DOCUMENT RESUME

ED 213 587 SE 036 239

AUTHOR Siford, M.
TITLE A Study of Algebra in the Provincial High Schools' Curriculum. Report No. 20.
INSTITUTION Papua New Guinea Univ. of Technology, Lae.
PUB DATE Oct 81
NOTE 73p.
AVAILABLE FROM The Director, Mathematics Education Centre, Papua New Guinea University of Technology, P.O. Box 793, Lae, Morobe Province, Papua New Guinea (no price quoted).

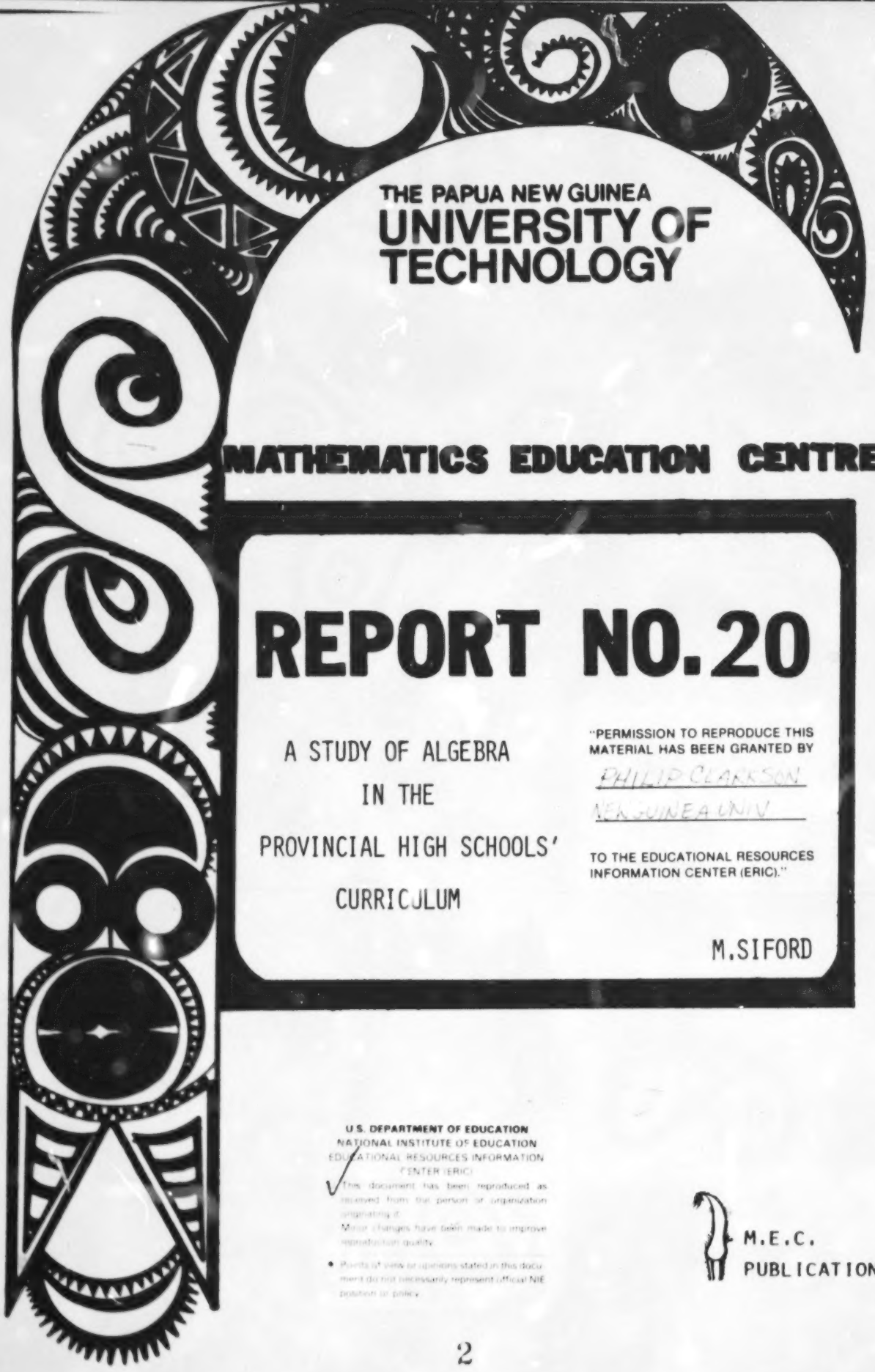
EDRS PRICE MF01/PC03 Plus Postage.
DESCRIPTORS *Algebra; *Curriculum Development; Educational Research; Equations (Mathematics); *Mathematics Curriculum; Mathematics Instruction; Problem Solving; Secondary Education; *Secondary School Mathematics
IDENTIFIERS *Mathematics Education Research; *Papua New Guinea

ABSTRACT

This report investigates recent tenth-grade algebra achievement, after first suggesting there are good reasons for maintaining algebra in the syllabuses for grades nine and ten. Results indicate the students studied have mastered such algebraic skills as "collecting like terms" and "substitution in formulae." It is noted that these pupils experienced great difficulty in framing problems in algebraic terms, and in accepting that letters can be used in a general sense and not just stand for one specific unknown number. It is argued that the present curriculum, which introduces algebra through the study of formulae and equations, produces this fixation and prevents students from developing a broader concept of the use of letters in mathematics. The document concludes with recommendations for curriculum development in algebra at the secondary level. Suggested activities are framed in a problem-solving context in which letters are used meaningfully rather than simply as objects to be manipulated. The approach attempts to focus on more general concepts of letters as variables to provide the foundation on which algebraic conventions and skills can be built. (MP)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED2135587



THE PAPUA NEW GUINEA
UNIVERSITY OF
TECHNOLOGY

MATHEMATICS EDUCATION CENTRE

REPORT NO. 20

A STUDY OF ALGEBRA
IN THE
PROVINCIAL HIGH SCHOOLS'
CURRICULUM

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

PHILIP CLARKSON
PNG UNIVERSITY OF TECHNOLOGY

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

M.SIFORD

U.S. DEPARTMENT OF EDUCATION
NATIONAL INSTITUTE OF EDUCATION
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

✓ This document has been reproduced as received from the person or organization originating it.
Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official NIE position or policy.

M.E.C.
PUBLICATION

SE 036 239

THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

THE MATHEMATICS EDUCATION CENTRE

The Mathematics Education Centre is a successor of the Mathematics Learning Project set up in 1972 with the aid of the Nuffield Foundation, the R.E. Ross Trust of Victoria and other philanthropic bodies. The establishment of the Centre was made possible with aid from the British Council over the period 1978 to 1979. From 1980 the Centre has been completely funded by the University of Technology.

The brief of the Mathematics Education Centre includes a substantial commitment to educational research at all levels of the education system. In addition the Centre provides advice and assistance to tertiary level institutions, the National High Schools, the Technical Division of the Education Department and to schools in the Morobe Province.

Research projects are undertaken by members of both the Centre and the Mathematics Department of the University. Visitors from overseas are also encouraged to work at the Centre on specific projects. This occasional series reports on the work conducted by resident staff and the visitors to the Centre.

Copies of M.E.C. reports and further information about the Centre is available from :-

The Director,
Mathematics Education Centre,
Papua New Guinea University of Technology,
P.O. Box 793,
LAE. MOROBE PROVINCE.
Papua New Guinea.

(A list of the Reports published and other M.E.C. publications is on the last page of this Report.)

REPORT NO. 20

A STUDY OF ALGEBRA IN THE PROVINCIAL HIGH SCHOOLS' CURRICULUM

M. SIFORD

M.E.C.
PUBLICATION

SERIES EDITOR
P. CLARKSON
OCT. 1981

Published and printed by

Mathematics Education Centre Press,
Papua New Guinea University of Technology,
P.O. Box 793,
Lae,
Papua New Guinea.

CONTENT

ABSTRACT 1

1 Some Aspects of Curriculum Development in P.N.G.

1.1 Background 2
1.2 The Place of Algebra in the P.N.G. Syllabus 4
1.3 The Mathematics Subject Masters' Survey 7
1.4 The Survey of Institutions 9
1.5 Summary 11

2 A Survey of Student Achievement in Algebra

2.1 A Classification of Letter Usage 11
2.2 The Algebra Test 14
2.3 Test Results - Student Strengths and Weaknesses 15
2.4 Test Results - An Algebra Hierarchy 18
2.5 Recommendations for Curriculum Development at the P.H.S. Level 20

3 Activities for Teaching Algebra in Grades 9 and 10

3.1 Sequence and Activities 25
3.2 Implementation 37

References 39

Appendices 41

EDITOR'S NOTE

This Report focusses on one area of the mathematics curriculum which is notoriously difficult for students no matter what country they are in, and yet is one important stepping stone to higher mathematics.

One of the factors highlighted in this report is the inability of post year 10 students to manipulate symbols in non-routine situations. This conclusion echoes comments made by Sullivan in Report 15 when he was discussing the difficulties Provincial High School students have in solving word problems with numerical solutions. Souviney (1981) has also noted the need for problem solving skills to be taught in the Community Schools. It would appear then that throughout the education system teachers can no longer simply teach basic mathematical skills, but they must also emphasize the use of such skills in 'non-routine' situations. However all of this implies the need for continued research at the classroom level, and more support for the teachers at the chalk-face through useful inservice work and the adequate supply of teaching materials. Unless an ongoing, consertive attempt is made to change the methods of teaching used in the classroom, teachers will continue to teach as they were taught.

The research reported here was not an M.E.C. project, but was carried out while Mr. Siford was Acting Director of the Mathematics Education Unit - U.P.N.G. Since the M.E.U. ceased to function at the end of 1980, it seemed best to publish the results of its major project as an M.E.C. Report.

SOUVINEY, R. 'Teaching and Learning Mathematics in the Community Schools of Papua New Guinea'. *Indigenous Mathematics Project Working Paper 20,* Department of Education, Papua New Guinea, 1981.

ABOUT THE AUTHOR

Mr. Mike Siford has been a lecturer at the University of Papua New Guinea since 1978, dividing his time equally between the Departments of Mathematics and Education. In 1980 he was seconded for one year by the Department of Mathematics to become the first Acting Director of the Mathematics Education Unit at University of Papua New Guinea.

ACKNOWLEDGEMENTS

The author wishes to gratefully acknowledge the assistance provided by Dr. Randall Souviney, who critically read the earlier drafts of the manuscript, and Mr. Phil Clarkson, whose help and encouragement was invaluable throughout the study, and who was instrumental in its publication.

A STUDY OF ALGEBRA IN THE PROVINCIAL HIGH SCHOOLS CURRICULUM

ABSTRACT

The recent and continuing rapid expansion of the secondary school system together with the growing problem of unemployed school leavers are reasons for a reappraisal of the school curriculum. One topic in the traditional mathematics syllabus is elementary algebra and this is subject, as are many other topics, to opposing demands. On the one hand students entering university are equipped with minimal algebraic skills and the universities demand higher entry standards; on the other, employers of grade ten leavers require numerical and measurement skills in preference to algebraic skills. The first part of this report suggests that there are good grounds for maintaining algebra in the syllabuses for grades nine and ten.

The second part reports the results of a survey of Preliminary Year Students at the University of Papua New Guinea. The results indicate that these students, who are recent grade ten leavers, have mastered such algebraic skills as 'collecting like terms' and 'substitution in formulae'. Few, however, are able to use such skills in non-routine situations. In particular these students experienced great difficulty in framing problems in algebraic terms, and in accepting that letters can be used in a general sense and not just stand for one specific unknown number. It is argued that the present curriculum, which introduces algebra through the study of formulae and equations, produces this fixation and prevents students from developing a broader concept of the use of letters in mathematics.

The third part provides recommendations for curriculum development in algebra at the secondary level. It attempts to focus on the more general concept of a letter as a variable to provide the foundation on which algebraic conventions and skills can be built.

The suggested activities are framed in a problem solving context in which letters are used meaningfully rather than simply as objects to be manipulated. It is thought that this approach will produce mastery of those skills which are developed by the current syllabus and, in addition, will foster the development of problems solving skills and the acceptance of a letter as a generalised number.

1 SOME ASPECTS OF CURRICULUM DEVELOPMENT IN P.N.G.

1.1 BACKGROUND

Mathematics curriculum development in P.N.G. has benefitted from major projects at all levels. From 1973 to 1976 the Mathematics Learning Project (Lean, 1975) produced materials for beginning students at the P.N.G. University of Technology and investigated different modes of teaching. In 1975 the Syllabus Advisory Committee of the Department of Education recommended modifications to the secondary mathematics curriculum, and since that time there has a UNESCO-UNDP secondary mathematics curriculum project charged with the development of appropriate syllabuses and curriculum materials (Britt, 1980). The Indigenous Mathematics Project (I.M.P.), which began in 1977, has conducted research into counting and classification systems in P.N.G. (Lancy, 1978). During 1980 the I.M.P. conducted a Curriculum Development and Trialling Study to provide guidelines for future curriculum development efforts at the primary level (Souviney, 1980).

Table 1 indicates the current state of development of mathematics material for the school system in P.N.G.

Major changes are about to take place in curriculum development efforts at both the primary and secondary levels. The I.M.P. will finish in 198[illegible] by which time the results of the 1980 Trialling Study will have been used to develop a set of consumable pupil workbooks for grade 3. Curriculum development for the primary schools will then be centred on a three year World Bank Project to write texts for grades 4, 5 and 6. There has been a close liason between the secondary and primary projects in an attempt to provide a consistent and systematic development of the content, and to use similar methodology. The World Bank Project will therefore be filling in between the materials produced for grade 3 and those in use at grade 7 and 8. Hopefully, the result of this co-operation will be to reduce problems at the primary-secondary interface.

SCHOOL	MATERIALS	COMMENTS
Community (Grades 1-6)	Syllabus and teacher's guide for all grades. Initial research now being undertaken to develop material for pupils.	Children aged 7 and above are eligible. Some 58% of 7-10 years old attended school in 1979.
Provincial Highs (Grades 7-10)	Syllabuses and teacher's notes are available for all grades. Pupils booklets for grades 7 and 8 are now in schools.	About one third of grade 6 leavers proceed to P.H.S. Selection is by a quota system. At the end of grade 8, 40% are excluded.
National High (Grades 11-12)	Syllabuses are available. Schools adopt a variety of overseas texts for classroom use. The production of detailed teachers notes for the syllabus began in 1980.	13% of students move from P.H.S. to N.H.S. Places are allocated on results of generalized test held midway through year 10.

TABLE 1: Summary of Curriculum Development in P.N.G. Schools.

The UNESCO-UNDP secondary mathematics project ceased at the end of 1980. Materials produced in booklet form for grades 7 and 8 have been compiled and edited for re-issue in text book form; grade 7 in 1981 and grade 8 in 1982. Plans to develop texts for grades 9 and 10 under the direction of the same writer, Britt, are currently being formulated. It appears likely that such direction will come from overseas, and that the syllabus advisory committee will therefore be required to adopt a more active role in the curriculum development process.

This brief outline of the trends in the development of mathematics curricula in P.N.G. indicates an extremely fluid situation over the next few years. It therefore seems essential to identify guidelines for the future to ensure that development continues in a coherent fashion. The results of the I.M.P. Curriculum Development and Trialling Study will provide direction for the World Bank Project. During 1980 there was a comprehensive evaluation of the grade 7 and 8 pupils booklets and the way in which these are being used in the schools (Hayter, 1980). Hayter's report provides much useful information for those concerned with the development of texts for grades 9 and 10. However, research is also required to identify and overcome problems with the teaching and learning of specific topics in the syllabus. One such topic is Algebra which is the subject of this report.

1.2 THE PLACE OF ALGEBRA IN THE P.N.G. SYLLABUSES

The present grades 7 and 8 syllabuses were first issued to schools in 1976 together with trial syllabuses for grades 9 and 10. Modified versions of the grades 9 and 10 syllabuses were issued in 1978, and these remain in use today. Appendix A represents a summary of the algebra included in the Provincial High School syllabuses. These documents provide an outline of objectives, content, and teaching suggestions, and are in effect a combined syllabus and teachers guide.

The writing and trialling of pupil's booklets for grade 7 took place in 1976 and 1977. These materials were then modified and introduced to all schools for use in 1978. Unit 7.14 - *Number Problems* consisted of 16 pages and was entirely based on number machines. These were used to provide a concrete context for the gradual introduction of 'letters standing for numbers' as a kind a mathematical shorthand; the letter i being used to stand for *input number* and r to stand for the *output number*. Grade 8 pupil's booklets were developed during 1979 and final versions reached all schools for use in 1980. The materials produced did not include a booklet for Unit 8.14 - *Problems and Relations*, which was originally seen as an extension of Unit 7.14. The syllabus for grade 8 indicates that Unit 8.14 uses the concepts of number machines and arrowgraphs to generate number pairs, and that

the relationship between the first and second numbers can be illustrated on coordinate graphs.

During 1980 the set of grade 7 pupil's booklets were modified and rewritten in the form of a two volume mathematics text. These texts will be issued to schools for use in 1981. Significantly the unit on number problems has been omitted from the texts. The existing syllabuses have therefore been modified by the introduction of these newer curriculum materials. The grades 7 and 8 texts do however contain much more work with whole numbers and with decimals than did the pupils' booklets. This gain has been at the expense of effectively deleting Units 7.14 and 8.14 from the syllabuses.

Algebraic topics have therefore been removed from the curriculum for grades 7 and 8. Unit 9.6 - *Numbers and Symbols 1* will be the pupil's first real introduction to algebra. The syllabus notes suggest the use of number machines to investigate 'order of operations', and letters are introduced by means of 'fruit - salad algebra'. This is perhaps unfortunate. Such shorthand devices, as $3b + 2b = 5b$ standing for 3 bananas + 2 bananas = 5 bananas, present a concept of a letter standing for an object. As this stage the concept of a letter standing for a number, and the idea that letters used in this way obey the number laws, are of prime importance. The present grade 9 syllabus was written in 1978 before the above changes took place. Unit 9.6 can no longer build on the content of the former Units 7.14 and 8.14 and suitable modifications to *Numbers and Symbols 1* need to be made.

Before examining the specifics of content and methodology, it is appropriate to review the state of the educational system to appreciate the significance of algebra in the secondary schools. In general, Community School teachers are poorly trained and lack the materials and in-service training support required to provide effective instruction (Roberts, 1981). The quota system, which provides all Community Schools with the opportunity to produce grade 7 entrants, together with poor instruction at the primary level ensure that the majority of pupils entering High School have a poor knowledge of mathematics. At the end of grade 8, some 40 percent of pupils are excluded from school. Given these circumstances, it is reasonable to assume that algebra should not be included in the syllabuses for grades 7 and 8.

There are widespread reports from employers and post-secondary institutions about the mathematical shortcomings of the present grade 10 leavers (Roakeina, 1977; Jones, 1980). These reports indicate that students are not proficient in such topics as operations with decimals, fractions, ratio and proportion, percentages, estimation, measurement and so on. More recent evidence (Sullivan, 1981) suggests that the real problem lies less in these basic skills but more in the understanding of concepts and applications. Whereas it could be argued that algebra has no place in grades 9 and 10 until such time as these short-comings are remedied, the universities identify the lack of algebraic skills as a major handicap at the tertiary level. In a review of the National High School syllabuses in 1980, the mathematics lecturers at the University of Technology made the following statement.

> *'The main weakness, and the most disturbing, is in algebra. Simple conventions such as the difference between $-x^2$ and $(-x)^2$ are hardly understood at all. Very few students are able to cope confidently with expressions like*
>
> $\frac{a}{0}$, $\frac{0}{a}$, $1 \div \frac{1}{a}$, a^{-1}, $a^{\frac{1}{2}}$ *etc.*
>
> *Setting out of work is often haphazard. More importantly, this often results in incorrect working. For instance, brackets may not always be used where they should be, so that $(24x + 20)(16x^2 - 5x + 2)$ will be written as $24x + 20 (16x^2 - 5x + 2)$. This indicates a lack of understanding of the purpose of brackets. (It happens too often to be regarded as carelessness). Few students can identify and cancel correctly common factors in the numerator and denominator of fractions. Another area of concern is in their understanding of functions. It is felt that this unit is particular is a bit ambitions in its content.'*

These views are supported by the mathematics staff at the University of Papua New Guinea who are experiencing similar problems in their teaching.

Against this background, it was decided to further investigate the appropriateness of algebra in the grades 9 and 10 syllabuses. Two surveys were conducted. The first was used to find out what teachers think about the appropriateness of teaching algebra at the Provincial High School level. The second sought the views of employers

of grade 10 leavers on this issue. These surveys and their results are described in the next section.

Algebra must also be considered in relation to the other components of the mathematics curriculum. In recent years there has been a growing awareness of an imbalance in mathematical curricula in which pupil mastery of computional skills has not been accompanied by the ability to use such skills in situations which require their application. American educators have called for due attention to problem solving in the 80's (N.C.T.M., 1980), and this position has been endorsed by P.N.G. teachers at the Third Mathematics Groups Workshop held at Goroka 24 - 28 November, 1980. The development of syllabus materials to aid the teaching of algebra should therefore include suitable problem solving activities.

1.3 THE MATHEMATICS SUBJECT MASTERS SURVEY

Britt conducted a survey of mathematics subject masters in Provincial High Schools to collect data relating to the grade 9 syllabus. The questionnaire used is shown in Appendix B. Completed questionnaires were received from 40 of the total of 100 schools. The data relating to the three units on algebra is summarised in table 2.

The results of the survey indicate clearly that teachers believe they can cope with the algebra presently included in the grade 9 syllabus. It also suggests that teachers are happy with the performance of their students on this topic. The application of equations to the solution of simple problems appears to be the most difficult objective to achieve.

SECTION A

UNIT TITLE	OBJECTIVES	Planning and Preparing lesson to achieve objective is			Pupils achieve objective with		
		Not Diff.	Mod. Diff.	Diff.	No Diff.	Some Diff.	Great Diff.
NUMBER PATTERNS	9.1.-recognise and continue patterns of numbers.	28	7	0	15	22	0
NUMBERS AND SYMBOLS 1	9.6.1-numerical problems with two operations.	27	10	0	21	17	1
	9.6.2-use of math shorthand:like and unlike terms.	26	9	2	21	14	4
	9.6.3-write math. formulas in short-hand notation.	28	8	0	18	17	4
	9.6.4-use formulas to work out areas and volumes.	33	5	0	16	21	3
	9.6.5-evaluate algebraic expressions.	30	7	0	10	20	10
EQUATIONS	9.12.1-solve simple number sentences with decimals and fractions.	25	7	0	18	12	2
	9.12.2-write English sentences as math. sentences.	20	10	1	12	13	8
	9.12.3-solve linear equations involving not more than two steps.	22	9	0	9	18	5
	9.12.4-solve simple word problems.	19	9	2	4	15	12

SECTION B

'It is proposed to include some introductory work on directed numbers in grade 9 rather than leave it all to grade 10'. Responses to this statement were:

Strongly Agree	Agree	No comment	Disagree	Strongly Disagree
30	7	0	1	0

TABLE 2: Results from Subject Masters' Survey.

The following comments relevant to algebra were made in section B:

*Algebra - extend Unit 9.6.
*Inequations should be included with equations.
*Algebra is neglected and should be taught at this stage.
*Include work on indices and logarithms.
*Include the equations of lines.
*Include quadratic equations.
*Directed numbers will help in solving equations.
*Combine Unit 9.6 with Unit 9.12.
*Leave Equations to grade 10; it needs more than 2 weeks.

The following comments were made on Unit 9.1, *Number Patterns*.

*See no point in Pascal's Triangle and Fibonacci.
*Number patterns are done in grades 7 and 8 and there is little new work in grade 9. This could be done earlier.

1.4 THE SURVEY OF INSTITUTIONS

In June 1980 a conference for the Heads of Tertiary Education and Training Institutions was held to discuss the 'Problems and Standards of Entry to Post Secondary Education and Training Institutions in the 1980's'. This conference led to the establishment of a Post-Secondary Mathematics Group consisting of representatives from training schools from both government and private sectors. The survey shown in Appendix C was sent to 24 institutions, and replies were received from the 12 respondents listed in Appendix D. The summary of these responses is shown in Table 3.

The results in Table 3 indicate that the use of both positive and negative indices (in scientific notation), the ability to substitute in formulae, and the solution of linear equations were considered to be the most essential topic. Algebraic fractions, fractional indices, and the solution of simultaneous and quadratic equations were considered to be the least essential topics. A comparison of Table 3 with Appendix A shows that the algebraic content of the grades 9 and 10 syllabuses closely matches the requirements of employers.

The reply from the Burns Philp Training Manager was most interesting. Having ticked each topic as 'not required', he went on to comment:

> *Our response would tend to be misleading - but in the context of your question there is no area of our present business activity where a grade 10 recruit would be required to employ algebra in the course of his/her duties.*

CONTENT	ESSENTIAL	USEFUL	NOT REQUIRED
1 (a) Collection of like terms	3	5	3
(b) multiplying out brackets	4	2	4
(c) factoring expressions	3	4	4
(d) algebraic fractions	0	4	6
2 (a) positive indices	8	1	2
(b) negative indices	5	1	5
(c) fractional indices	2	1	8
3 (a) simple substitution in formulae	9	1	1
(b) substitution requiring more difficult manipulation	8	1	2
4 (a) solution of linear equations	7	2	2
(b) solution of quadratic equations	1	5	5
(c) solution of simultaneous equations	2	5	4
(d) to set up an equation given a problem in the work situation	3	5	3

The statement, 'Algebra should be included in the grades 9 and 10 syllabuses' produced the following response:

Strongly Agree	Agree	No Comment	Disagree	Strongly Disagree
8	4	0	0	0

TABLE 3: Results from the Survey of Institutions

> *We do however employ graduates in mechanical engineering and agriculture for whom skill in algebraic equations in particular is essential.*
>
> *Generally we support the continued inclusion of algebra in grades 9 and 10.*

This statement is in agreement with work done in England. In a study of the mathematical needs of sixteen year old school-leavers entering employment, Fitzgerald (1980) compared commerical trainees with engineering apprentices. He writes that the mathematical demands made on commerical trainees are much narrower in scope. There appears to be little or no algebra nor, as one would expect, goemetry or trigonometry, some elements of which are required by engineering apprentices. However, he warns against allowing pupils to choose a narrow vocational oriented mathematics syllabus since this could seriously limit their future choice of career.

1.5 SUMMARY

The content of the existing grades 7 and 8 syllabuses has been modified by subsequent curriculum development with the result that the study of algebra now begins at the grade 9 level. This is seen as an appropriate development. The inclusion of algebra in the grade 9 syllabus is supported by subject masters who feel that both they and the pupils can cope with the topic, and by employers and others who consider that the use of indices, formulae and equations are essential pre-requisities for their own training programs. Unit 9.1 - Number Patterns is not considered relevant by some teachers.

Part 2 investigates the algebraic skills of recent grade 10 leavers and identifies their strengths and weaknesses. It is suggested that student weaknesses result from the content of the present algebra units; ways in which these units can be restructured and improved are suggested.

2 A SURVEY OF STUDENT ACHIEVEMENT IN ALGEBRA

2.1 A CLASSIFICATION OF LETTER USAGE

In algebra, letters are used in a number of different ways. Anderson (1978) points to one of the fundamental differences.

> *Perhaps it is in order to point out here a distinction... between problems in equation-solving which involve letters standing for particular but unknown numbers, eg. to solve* $3x + 7 = 16$*, and descriptions of the "arithmetical" structure of the real number system in which letters stand for arbitrary and general numbers, e.g.* $x(y+z) = xy + xz$*. The symbolization and the use made of letter symbols, through outwardly alike, are different in kind; (p2).*

To distinguish these two cases, the terms specific unknown number (S.U.) and generalized number (G.N.) will be used. In addition, there are instances where numbers and letters are used largely in a computational sense, and others where some sort of comprehension is an additional

requirement. Küchemann (1978) has defined six levels for describing different ways in which letters may be used. The six levels described below are a modification of Küchemann's classification. Examples are given for each level.

LETTER EVALUATED (L.E.). The numerical value of the letter can be directly determined by substitution, simple trial and error, or the use of numerical rather than algebraic skills.

* What is the value of m if $m-7 = 12$?
* Given $y = 3x + 4$, find y when $x = 6$.

LETTER IGNORED (L.I.) There is no need to either manipulate or evaluate the letter. Items can be completed using numerical skills only.

* If $a + b = 43$, then $a + b + 2 = ...$
* Write down the smallest and largest of these:
  $n+1$, $n+4$, $n-7$, n , $n-3$ smallest... largest .....

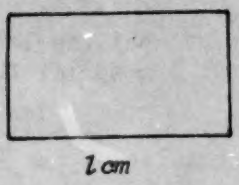
LETTER AS AN OBJECT (L.O.) Letters are manipulated according to known rules. These is no requirement to understand what the letter stands for - it is simply treated as an object to be operated upon.

* Simplify $4pq^2 \times 5p^3r$
* Expand $4(3m - 2)$
* Factor $3xy - 12yz$
* Make l the subject of the formula $A = lb$.

LETTER AS AN OBJECT WHICH IS SUBSEQUENTLY EVALUATED (L.O.E.)
The letter is firstly manipulated and then evaluated.

* Solve the equation $4x - 7 = 2(x+3)$
* Given $T = 3m\sqrt{\frac{x}{y}}$, find y when $T = 100$, $m = 7$ and $x = 4$.

LETTER REQUIRING COMPREHENSION AND/OR CHOICE OF OPERATION (L.C.)
Letters are used in the solution of problems which require an understanding of what the letter stands for, and may call upon the learner to choose the appropriate operations on the letter.

* 

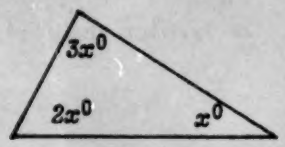
What is the area of this rectangle?

*

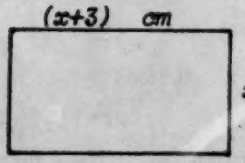
What is the total length of this piece of string?

* Mary has P marbles and John has Q marbles.
How many marbles do they have altogether?

LETTER REQUIRING COMPREHENSION AND EVALUATION (L.C.E.)
This is an extension of the L.C. class and requires the eventual evaluation of the letter.

* 

Find the size of the largest angle of this triangle.

* 

The area of this rectangle is $70cm^2$.
The width of the rectangle is xcm.
Calculate the length of the rectangle.

NUMERICAL ITEMS (N.) are those that do not involve the use of letters.

The grades 7-10 syllabuses have an emphasis on number, measurement and statistics and include some geometry and algebra. Algebraic topics (see Appendix A) concentrate on the simple idea of a letter standing for a number and the development of routine manipulative skills. The following classification was therefore designed in an attempt to form a hierarchy of learning tasks. It was thought that L.C. and L.C.E. items, which require an understanding of what the letters represent, would present the greatest difficulty to students since little work of this nature is included in the syllabuses.

L.E. items which require computation and not
L.I. algebraic manipulation

L.O. items which require routine algebraic manipulation
L.O.E. and numerical skills.

L.C. items which require an understanding of what the letters stand
L.C.E. for and the ability to operate on them appropriately.

The classification will be discussed with the results of the algebra test in a later section.

2.2 THE ALGEBRA TEST

Part 1 gave a brief outline of the place of algebra within the mathematics curriculum. In order to gain more detailed information about the difficulties experienced by students, a survey of students at the University of Papua New Guinea was conducted in October 1980. In part 3 of this report, the conclusions drawn from this survey are used to develop guidelines for curriculum development for grades 9 and 10.

A total of 150 Preliminary Year students, 122 from the main campus and 28 from Goroka Teachers College, were tested using the survey shown in Appendix E. The test was not compulsory with the result that only 122 out of a total of 160 main campus students took the test. The mathematics lecturers reported that those who did not take the test were generally very weak students, who would not have been expected to do well. The Goroka group consisted of students who had elected to take mathematics as one of their teaching subjects. The students were in the last weeks of their Preliminary Year course and would therefore have received an extra years instruction in mathematics compared with grade 10 leavers.

The test was developed from the work of Küchemann (1978) which is also reported by Hart (1980). The test was designed to give information on the types of question at which students had gained mastery, and to identify situations which led to failure. Küchemann's research focussed on the testing of concepts rather than rule acquisition, or the use of algebraic skills. The present work included a number of skill items such as multiplying out brackets, solving equations and factoring expressions as well as the majority of items used by Küchemann. There are a total of 57 questions or parts of questions of interest, and these items are shown in Appendix E using labels I 1.....I 57. Also shown in Appendix E is the classification of each item using the abbreviations L.E., L.I., L.O.,

L.O.E., L.C., L.C.E., and N. A first version of the survey was trialled using a group of 10 third year education students who assisted with problems of language, suitable task environments, and the time to be allowed for the test.

The test was marked using a 9-point code which is shown at in Appendix F. Using this method several frequently occurring wrong responses were identified and these are discussed below. The results at in Appendix G indicate the students' response for each of the 57 items under the 9 categories of the marking code. The data is presented as the percentage of students responding in the manner indicated. A total score for each item was obtained by adding the scores for categories 1 and 2 to half the score of category 3. This total score is shown as a percentage in Appendix E thus (94.3). The performance of the Goroka students was at the same overall level as the Main Campus group and generally followed the same pattern of response. The results for both groups have therefore been combined.

2.3 TEST RESULTS - STUDENT STRENGTHS AND WEAKNESSES

(1) The majority of preliminary year students mastered specific skills such as evaluation and the routine manipulation of symbols. The following items illustrate this point.

* remove brackets and simplify $(x-4)(x+4)$ (93.3)
* factorize $y - 2y - 15$ (78.7)
* What can you say about m if $m = 3n+1$, and $n = 4$ (90.7)

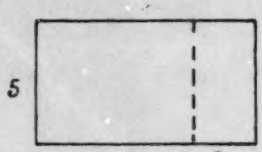
Some students however had difficulty with conventions. Some common misunderstandings were:

* $2a + 5b = 7ab$ (15.3)
* $J + T = JT$ (10.0)
* $(a-b) + b = ab - b^2$ (14.7)

This last item is typical of many responses to question 2 in that the grouping symbols ( ) were often incorrectly associated with the operation of multiplication. Question 10 was also a source of confusion. Responses such as nm^2 and $xy^2 - 6^2$ were unsuccessful attempts to provide a unit of measurement with the answer.

(2) There were many instances where wrong answers were obtained by replacing a letter by a number (CODE 6). This seemed to arise when students were in a situation which they felt to be too difficult; the problem was "simplified" by arbitrarily replacing letters with numbers. Instances of this behaviour will be called NUMERICAL REGRESSION. Some examples are:

* If $e + f = 8$
then $e + f + g = 12$



A=35

P=16

(3) The algebraic items of questions 1, 10, 19, 21 and 25 were preceeded by a related numerical item, which will be called a NUMERICAL FILTER. The filters were useful in that they indicated ways in which students interpreted the problem. For example, in question 21 - item 50, 77.3% of students gave the "correct" response of K38, and 19.3% got the item "wrong". However more than half of the wrong answers (10% of the total responses) were K70 or K86. These answers are readily explained in terms of 4 hours of overtime EVERY DAY for a 5 or 7 day week respectively. Of the 77.3% of students who correctly answered the numerical filter, only 11.3% were able to repeat the same steps in the algebraic case. None of the students who responded K70 or K86 were able to produce an equivalent algebraic expression for the problem as they saw it.

Whilst the numerical filters were useful to the researcher as a means of identifying the ways in which students interpreted the problems, they did not appear to be useful to the students themselves. Students were unable to use the filters as a means of modelling the problem in a numerical case and then applying the same model to the algebraic case.

(4) There was a large range in the level of difficulty of items belonging to the class LETTER REQUIRING COMPREHENSION. The easiest items were those where the underlying concepts were

familiar to the students and no real decision step was required. Items of this type are the first three on perimeter in question 11. Such items essentially involved the same sort of routine manipulation required by items in the LETTER AS AN OJBECT class. Items which were not routine but involved planning a solution strategy, were much more difficult. Question 10 illustrates both easier and harder items. Questions 20, 21 and 24 indicate, considerable difficulty encoding information into algebraic terms.

(5) Students had difficulty with items where the letters were being used as generalized numbers rather than specific unknown numbers, and often this led to numerical regression or incomplete solutions. Questions 8, 9, 11 (item 34), 15 (item 42) and 24 are examples of this behaviour. C = 4(22%) was a frequent particular solution to item 42, whereas C = 1, 2, 3, 4 (10%) and C < 5 (9.3%) were less popular but more general solutions.

(6) Item 46 produced the unexpected response H (4.7%) instead of the correct answer K-3(58.0%). Clearly H was produced by the process of counting back three letters of the alphabet from K. Whilst this is an incorrect response, it does perhaps provide an illustration of the ability to generalize, which is fundamental to the study of algebra.

Item 38 (5.3%) was the most difficult item on the test. Very few students translated 'what can you say about b if $b + 3 = 3b$' into 'solve the equation $b + 3 = 3b$'. Two major difficulties were the inability to translate the 'is equal to', and the fact the 3 occurred twice. Many students remarked that '$3b$ means $3 \times b$' so the equestion is wrong! Whereas an item such as 'solve the equation $x + 5 = 3x$' would have produced a high percentage of correct responses, item 38 proved to be a surprise. This kind of situation produces difficulties in trying to establish a hierarchy as will be discussed in the next section.

(7) In summary, a large majority (70 - 80%) of students were able to apply their knowledge and skills to routine situations such as equation solving and the use of formulae. A minority of students (about 30%) were able to use algebraic skills in situations requiring comprehension and the ability to develop a solution strategy.

2.4 TEST RESULTS - AN ALGEBRA HIERARCHY ?

Each item on the test was assigned to one of the seven classes L.E., L.I., L.O., L.O.E., L.C., L.C.E., and N. Appendix H summarizes this classification and provides a score for the mean degree of difficulty for each class. Clearly there is a very uneven distribution of items among classes and no firm conclusions can be drawn from such data. The data does however support the general statement made under part (7) of the previous section.

There are several factors which prevent the formulation of a rigid hierarchy and some of these are outlined below. Davis, Jockusch and McKnight (1978) have identified several classes of cognitive behaviour in relation to the learning of algebra. Two of these classes are Goal Orientation (G.O.), and Long Execution Sequences (L.E.S.). Item 35, factorize $am + bn - mb - an$, provides an example where students lacked goal orientation. A partial solution of $a(m-n) - b(m-n)$ or similar expressions was obtained by 17.3% of students, whereas only 7.3% took both steps to complete the item. Many students attempted to solve the equation $(5x+1)^3 + 5x = 349$ by expanding the left hand side. This produced a long execution sequence which few could master; it also led to a loss of goal orientation in that the right hand side was totally ignored. Another example of a L.E.S. is in item 44, simplify $\frac{2}{x+1} - \frac{x}{x^2-1}$.

Another factor to be considered is Acceptance of Lack of Closure (A.L.C.). This arises in situations where a "complete" answer cannot be obtained and the problem is in some sense not fully resolved. (UNESCO-UNICEF, 1974; Küchemann, 1978).

Hence item 14, add 4 to $3n$ (76.7%), is more difficult than item 16, multiply $3n$ by 4 (93.0%). For item 14, 18.7% of the students obtained "closure" by giving the incorrect response $7n$. Item 22, if $e + f = 8$ then $e + f + g = ...$, produced A.L.C. difficulties. The correct answer of $8 + g$ was closed to $8g$ by 2.7%, whereas 25.3% of students resorted to numerical regression to produce answers in the range 10 - 16.

Af Ekenstam and Nilsson (1979), have identified several difficult steps in the learning of algebra and geometry. Their investigation was carried out among the better fifth of 16 year old Swedish children, and showed that superficially similar items produced different levels of difficulty. For example, simplify $\frac{a}{a^2}$ and simplify $\frac{a^2}{a}$ were successfully answered by 36% and 87% of students respectively. In their discussion the authors comment:

> *Moreover lack of understanding appeared on essential points, e.g., that a letter stands for a number. It seems probable that almost every student would have given the correct answer to the problem "Write in lowest terms $\frac{15}{15}$" but only about half of the students mastered $\frac{a}{a}$.*
>
> *Many pupils had more or less mechanically learned how to find the solution to the equation $3x = 4$ without understanding what operation they had done, division by 3 (or multiplication by $\frac{1}{3}$) or even connect the method of getting a solution, with an operation.*
>
> *The fact that letters stand for numbers and that the same operations can be performed did not seem to be evident to all. However we are dealing here with essential conceptional understanding without which insuperable barriers to further studies in mathematics and subjects using mathematics, e.g. physics, must arise. (p 64).*

A factor for note here is the interpretation of letters as specific unknown numbers compared with generalized numbers. Item 42 provides an example. The responses were C < 5 (9.3%), C = 1, 2, 3, 4 (10.0%) C = 4 (22.0%). The total score of 30.3% assigned to this item masks these qualitative differences in response.

A final factor of interest is the numerical complexity (N.C.) of the task. The following equations illustrate this difficulty.

$$A + 4 = 10$$
$$A + 2.84 = 7.66$$
$$A + 1\frac{2}{3} = 4\frac{1}{4}$$
$$A + 6.04 = 3.91$$

As Af Ekenstam and Nilsson have pointed out, pupils often solve equations without understanding what operation they have done. However even if the essential operation of subtraction is understood in the above cases, the numerical complexity of the later items may lead to incorrect responses.

The abbreviations shown thus ( ) at in Appendix H refer to the factors G.O., L.E.S., A.L.C., G.N., and N.C. described above. There are undoubtedly other important factors which have not been identified here. Whilst this research has not identified a learning hierarchy for algebra, it is felt that the seven classes used, together with the five additional descriptors, provide a useful method of classifying algebraic items.

2.5 RECOMMENDATIONS FOR CURRICULUM DEVELOPMENT AT THE P.4.S. LEVEL

Traditional or school algebra is essentially the study of generalizations which may be expressed in words, and by the use of symbols and formulae. Further discussion of this issue may be found in Anderson (1978), Norris (1973) and Dieudonné (1972). The ability to read algebra, to appreciate the meaning of an equation or a graph and to associate it with its applications, is essential in a wide variety a situations, especially the learning of science and technology. Given that mathematics is the "language of science", there are two central aims in the teaching of mathematical symbolism. These are to be able to:

* Use symbols as an aid to communication; this requires both the ability to read (decode) and write (encode) mathematical symbolism.

* Manipulate symbols according to mathematical laws without having to think about the meaning of the symbols at the same time.

The achievement of both aims will permit the further study of science and technology and provide algebraic tools for use in problem - solving. The results of the Algebra Test indicate that the majority of students have difficulty with encoding information, and have developed the mind-set 'a letter stands for a fixed number'. Manipulative difficulties are less common and often arise from a lack of understanding of algebraic conventions. Language is an ever present problem.

Souviney (1981) has provided the following description of Polya's general problem solving plan.

Step 1 - Understanding the problem
Step 2 - Designing a solution strategy
- *(a) Guesstimation*
- *(b) Working backwards*
- *(c) Reduce to a simple case*
- *(d) Conduct an experiment*
  - *(i) Design a physical model*
  - *(ii) Make a sketch or diagram*
  - *(iii) Collect data and organise it in a table*
  - *(iv) Construct a graph*
  - *(v) Search for a pattern*
  - *(vi) Develop a general rule (function)*
- *(e) Add elements to the problem situation*

Step 3 - Carrying out the strategy
Step 4 - Evaluating the results

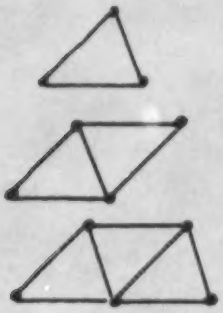
The use of the scientific method as suggested in step 2(d) provides a model relevant to the understanding of student difficulties in algebra. This is shown below, together with a description of a typical algebraic task.

A: MODEL OF THE SCIENTIFIC METHOD	B: TYPICAL ALGEBRAIC TASK
Conduct an experiment using materials relevant to the learner's environment.	
↓	
Simulate the experiment using diagrams.	
↓	
Organize data in a table (or draw a graph).	
↓	
Search for a pattern, rule or relationship.	
↓	
Verbalize the pattern, rule or relationship.	Provide a verbal description of a situation or relationship.
↓	↓
Express the pattern, rule or relationship symbolically.	Require that a suitable symbolic model be produced.

Not every step in A is present in every learning task of this type, but a gradual progression through the stages *concrete → representational → symbolic* provides a meaningful context for the encoding process. This is in contrast to B where a word problem often presents comprehension difficulties which prevent successful encoding. In P.N.G. English, the language of instruction, is at best the learner's second language. It is therefore essential to minimize comprehension problems in order to facilitate the development of encoding skills. This can be done by designing activities, which involve the manipulation of familiar objects and the drawing of diagrams, to provide a basis for the symbolic work to follow.

Many students have difficulties with the concept of a letter as a generalized number. This type of restrictive thinking may be a result of the syllabus which has an emphasis on equations and substitution in formulae. It is therefore suggested that the study of functions is a more appropriate starting point since this topic focusses on letters used as generalized numbers rather than specific unknowns. As an example

consider the following experiment with triangles made of matchsticks and plasticine, or triangles drawn on dot paper.



Number of triangles	Number of dots	Number of Matches
1 →	3	3
2 →	4	5
3 →	5	7
10 →		

This experiment generates data suitable for tabulation, and the search for a pattern and a rule can begin. Usually the pattern is easier to see than the rule, as the above example demonstrates.

Number of triangles	Number of matches
1 →	3
2 →	5
3 →	7
4 →	9
5 →	11
10 →	
100 →	?

To form each additional triangle an extra 2 matches are required. This can be readily seen from the model, and from the *pattern* in the second column which increases by 2 at each step. The *rule* however directly connects the number of matches with the number of triangles.

Using the *pattern* to find the number of matches required to build 1000 triangles is clearly a more time-consuming task than using the *rule* "multiply by 2 and then add 1".

As noted above, some students have difficulty with algebraic conventions. For the grade 9 and 10 population as a whole such difficulties are likely to be more widespread; the students tested were only those grade 10 leavers selected for U.P.N.G. Such conventions should therefore be introduced slowly, and because of the special language problems that exist in P.N.G. great emphasis should be placed on verbalization. Hence the results of the above experiment should initially be expressed using

words only. "The number of dots is always two more than the number of triangles". A second stage is to introduce = and numerals to get "The number of dots = the number of triangles + 2". A third and perhaps much later stage might be *"d = t + 2"*.

Instone (1980) working with the School Mathematics Project on algebra for the 11 - 16 year olds, has reported the importance of introducing conventions very slowly. In the above example, translation of the rule "The number of matches is twice the number of triangles, plus one" might go through the following stages.

The number of matches = (2 × the number of triangles) + 1
$m = (2 \times t) + 1$
$m = (2t) + 1$
$m = 2t + 1$

It is therefore recommended that conventions be introduced slowly and that words and algebraic symbolism be used contiguously throughout grades 9 and 10.

Many students use numerical regression in an attempt to cope with algebraic problems which they find difficult. However students do not appear to take advantage of numerical filters to solve equivalent algebraic problems. Research is needed to find out if students can learn to develop their own numerical filters (or numerical models) as an aid to the solution of algebraic problems. This ability to encode data into a meaningful algebraic form appears to be not well developed at this time.

The next and final part of this report provides a detailed framework for the development of curriculum materials for grades 9 and 10. This framework has been designed to overcome the difficulties of encoding, the use of letters as generalized numbers, algebraic conventions and language.

3 ACTIVITIES FOR TEACHING ALGEBRA IN GRADES 9 AND 10

3.1 SEQUENCE OF ACTIVITIES

The suggestions made here are drawn largely from the work of Giles (1979), Dienes (1964(a), 1964(b)), Laycock and Schadler (1973) and materials written by the P.N.G. U.N.E.S.C.O. - U.N.D.P. MATHEMATICS PROJECT and the U.N.E.S.C.O. - U.N.D.P. CURRICULUM DEVELOPMENT UNIT AT U.S.P. 1970 - 1975. The sequence of activities is broken down into a number of stages which can be fitted into appropriate places in the existing syllabuses.

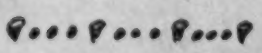
Stage 1

Activities suitable at this stage are those which require the manipulation of familiar objects and the drawing of diagrams. One such activity is to make a necklace using 1 shell and then 3 seeds as follows.

1 part

2 parts

3 parts



4 parts

5 parts

Number of Parts	Number of Shells	Number of seeds	Total Number of shells and seeds
1	2	3	5
2	3	6	9
3	4	9	13
4			
5			

The objectives for stage 1 are for pupils to be able to:

* Continue the activity by making further models (or diagrams).
* Complete and extend the table of data by recognizing the pattern of numbers down each column.
* Work out the rule that connects the number of parts with the number of shells (or seeds or the total).
* Check that the rule works in some additional cases.
* Describe the rule in words.
* Describe the rule in words using some abbreviations.

Hence in the above example the patterns are:

Number of shells of	Number of seeds	Total Number of shells and seeds
2	3	5
3	6	9
4	9	13
(ADD 1)	(ADD 3)	(ADD 4)
5	12	17
6	15	21

Whereas the corresponding rules are:

the number of shells is equal to the number of parts plus one
the number of seeds is equal to the number of parts multiplied by three
the total number is equal to four times the number of parts plus one

which are shortened to:

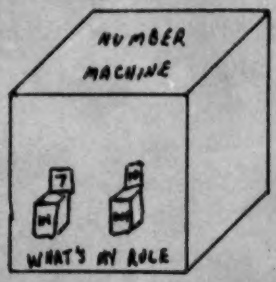
number of shells = number of parts + 1
number of seeds = number of parts x 3
total number = (4 x number of parts) + 1

Stage 2 - General

The aim of this stage is to be able to complete the process of translation from words into mathematical symbolism. Of great value here is the use of number machines, which provides an efficient means of generating data without the time-consuming process of modelling described in stage 1. Number machines can be used to demonstrate the use of brackets in arithmetical expressions, to introduce the idea of inverse operations, and as a systematic way to solve equations. Each substage is described below.

Stage 2A - A "Real" Number Machine

Ideally suited for this stage is a programmable calculator which can be used to generate data in a "what is my Rule" game type simulation of stage 1 activities. Otherwise a large box with an input and output pocket and a quantity of numbered cards can be used.



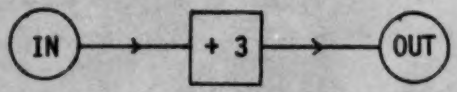
Here a pupil can insert a set of numbered cards one at a time into the input pocket, and the teacher (or another pupil) can place the corresponding card in the output pocket. At this stage only the teacher knows what the rule is; hence the following data may be generated.

input	output
7	10
5	8
11	14
2	5

Pupils can then guess the rule output = input + 3.

Stage 2B - Representation of a Number Machine

The concrete stage described above is next abstracted by the use of diagrams to represent number machines. The symbol ◯ may be used for input/output and □ for the machine rule. Hence the machine



Can be used to generate the data

RULE	+ 3
Input	Output
1	4
6	?
14	?
29	?

OR, reversing the process, the data

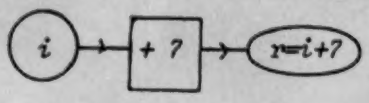
RULE	?
Input	Output
2	8
5	20
10	40
7	28

May be used to find the rule.

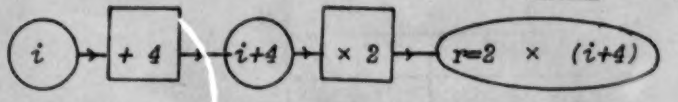
Exercises at this stage should use machines which require operations with fractions and decimals as well as whole numbers. Many examples are given in the grade 7 unit 14 pupils workbook.

Stage 2C - Introducing Letters

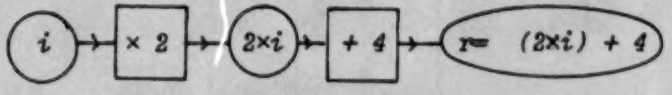
The introduction of letters as a further step in the translation from words to mathematical symbolism can now be made. Using number machines it is appropriate to use i for the input number and r for the output number or result. It is important to emphasize that, i and r stand for numbers. Hence a single machine may be shown:



The activities first encountered in stage 1 can now be completed. The necklace discussed above, page 25, leads to the expressions $S = p + 1$ and $e = 3 \times p$, where p stands for the number of parts, S the number of shells and e the number of seeds. The sentence "the total number is equal to four times the number of parts plus one" cannot be translated without the help of two machines. The combination of machines provides a natural illustration of the importance of using brackets for correct punctuation. Combining the machines $\boxed{+\ 4}$ and $\boxed{\times\ 2}$ we obtain:



one way, and



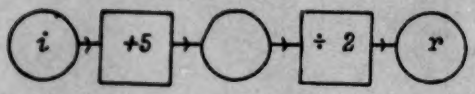
in another.

Choosing a variety of values for i, it can be shown that for example

$$2 \times (7 + 4) \neq (2 \times 7) + 4 \quad .$$

Suitable exercises at this stage are of the type:

(1) Complete the table of output numbers for this machine.



i	r
3	
7	
8	

(2) (a) Write down in words the rule for the following machine that connects the output number to the input number.
(b) Write down the rule using the letters r and i.

(3) Complete each table and find the rule of each machine.

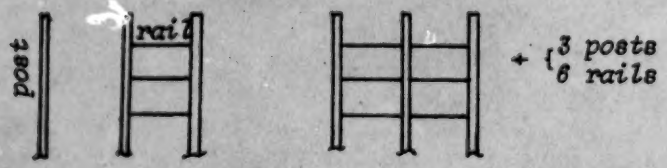
(a)

i	r
2	9
3	10
4	11
7	14
10	
12	

(b)

i	r
1	2
2	5
3	8
4	11
7	
11	

(4) The following diagrams show how to build a fence.



(a) Draw diagrams for 4,5 and 6 posts.
(b) Complete this table.

Number of posts	Number of rails
1	0
2	3
3	6
4	
5	
6	
10	
100	

(c) Let p stand for the number of posts and r for the number of rails. Write down the rule that connects the number of rails to the number of posts (i) in words (ii) using p and r.

Stage 20 - Using Letters Without Number Machines

Number machines are useful to efficiently generate data and to focus on the correct use of brackets to punctuate mathematical expressions. The introduction of algebraic conventions such as:

$2 \times i = 2i$
$2 \times (i+4) = 2(i+4)$
$(2i) + 1 = 2i + 1$

needs time and care, as these are the source of future difficulties. Perhaps this is best done when letters are used to write formulae in shorthand notation. Hence the formula for the area of a rectangle undergoes the following translation.

Area of a rectangle = length × breadth
$A = l \times b$
$A = lb$

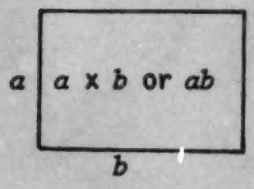
Materials such as match-sticks and drinking straws may be used to make models of many algebraic situations. This kind of activity depends on the concepts of area and volume and will be unsuccessful if these concepts are not well developed. Care is also required to ensure that the concept of *letters* standing for *numbers* is maintained; modelling activities should therefore include frequent evaluation by substitution. Some examples are shown below:

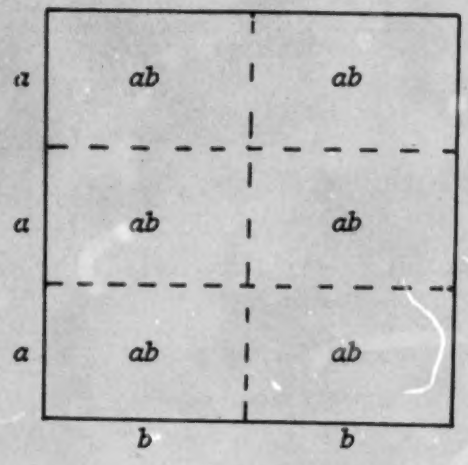
(1) a = _____ b = _____

_____ _____ _____ $a + a + a = 3a$

Evaluate $3a$ when $a = 4$ and $2b$ when $b = 7$.

(2) Area may be used to model multiplication:





$3a \times 2b = 6ab$

A rectangle is $3a$ cm wide and $2b$ cm long.
If $a = 6$ and $b = 4$, calculate $6ab$.
What is the area of the rectangle ?

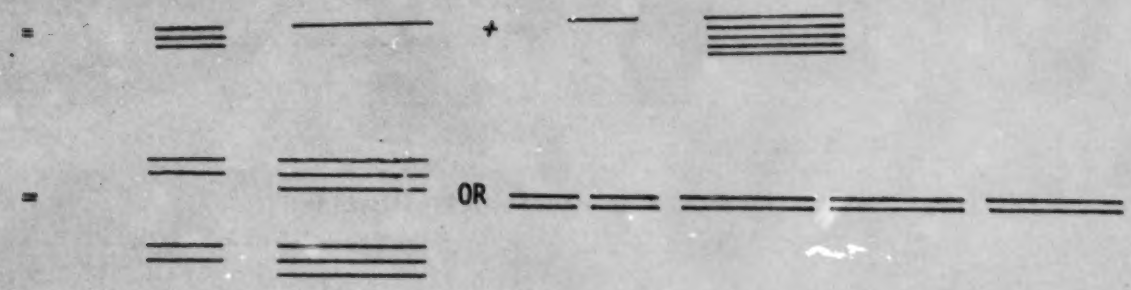
(3) Build a model of $3(a + 2b)$

_____ _____ _____
_____ _____ _____
_____ _____ _____

Complete the statements $3(a + 2b) = \ldots a + \ldots b$
$2(4a + 3b) = \ldots a + \ldots b$
$5(x + 6y) = \ldots x + \ldots y$

(4) This sketch shows how to collect like terms and simplify.

$(3a + b) + (a + 5b)$



$2(2a + 3b)$

Now put $a = 1$, $b = 4$ and evaluate (i) $3a + b$
(ii) $a + 5b$
(iii) $(3a+b) + (a+5b)$
(iv) $4a + 6b$

Are your answers to (iii) and (iv) the same ? Why ?

(5) d squared (area) and d cubed (volume) may be introduced with appropriate models. Pupils should build models of d, d^2, $2d$, d^3, $3d$, $2d^2$ etc and evaluate such expressions for several values of d.

Further evaluation exercises using familiar formulae (instead of models) will aid the recall of the conventions. Well known formulae such as $A = lb$, $A = 3.14R^2$, $P = 2(l + b)$ may be put into words before being evaluated. Other formulae may be used when proficiency with substitution has been gained.

Stage 2E - The Use of Graphs

The above discussion has been largely about verbal symbolism in an attempt to outline ways to translate a statement in words into an equivalent statement using letters. The statements:

"The number of shells is equal to the number of parts plus one"

"$S = p + 1$"

are essentially different verbal forms of communicating the same idea. The idea may also be presented visually in the form of a graph. Appendix A details the objectives of grade 8 unit 14 which are relevant here. These are to "discover relationships between sets of numbers ... and to illustrate simple relations on coordinate graphs".

With reference to the necklace example on page 25, above, the following coordinate graphs follow from the previous work.

Number of parts *(p)*	Number of Shells *(s)*
1	2
2	3
3	4
4	5
5	6

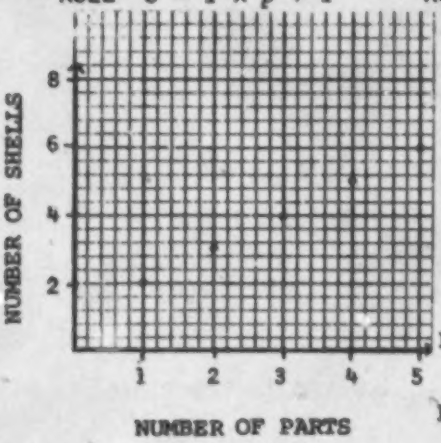
PATTERN ADD 1
RULE $s = 1 \times p + 1$

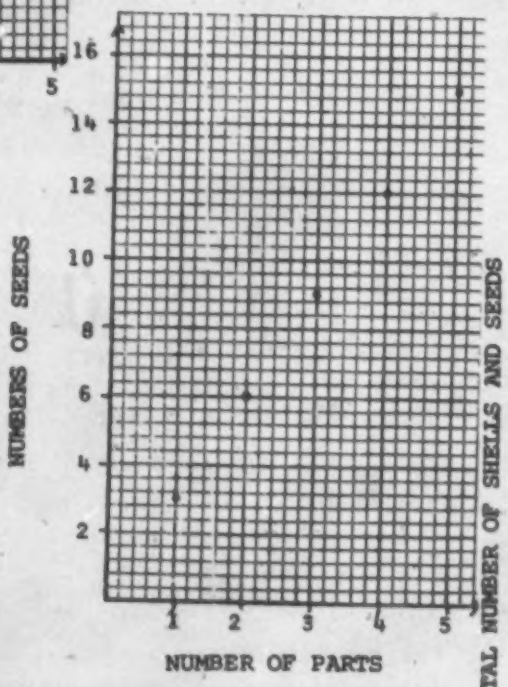
Number of parts *(p)*	Number of seeds *(e)*
1	3
2	6
3	9
4	12
5	15

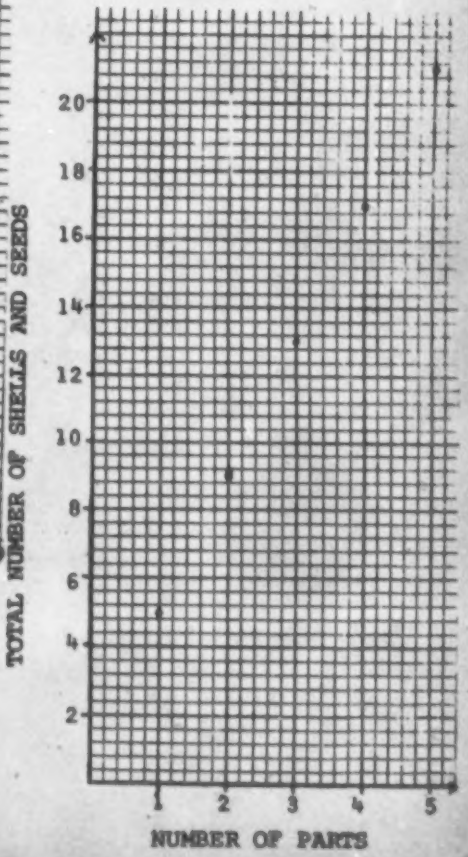
PATTERN ADD 3
RULE $e = 3 \times p$

Number of parts *(p)*	Total shells and seeds *(t)*
1	5
2	9
3	13
4	17
5	21

PATTERN ADD 4
RULE $t = 4 \times p + 1$







At present the equation of a line on the number plane is not included in the grades 7-10 syllabuses. However intuitive notions of the connection between the *pattern* and the *rule*, and the slope of the line should be developed at this stage.

Stage 3 - The Solution of Equations

Stages 1 and 2 focussed attention on the idea of functions and the use of letters to describe the rule which connects one set of numbers with another. Restricting the use of the letter to stand for one specific unknown number is now an easy step to take. A standard introduction is for the teacher to use a puzzle such as the following:

> Think of a number, multiply it by 2, add 18, divide by 2 and subtract 5. The pupils gives his answer and the teacher tells the pupil what number he started with by subtracting 4.

Perhaps a slightly better version of the puzzle is to plan for every pupil to finish with the same number; good arithmetic practice and easy to check! The teacher's explanation of the above puzzle is:

> I use k to stand for your number, and follow the instructions as given.
>
> | | | |
> |---|---|---|
> | Think of a number | - | k |
> | Multiply it by 2 | - | $2k$ |
> | Add 18 | - | $2k + 18$ |
> | Divide by 2 | - | $k + 9$ |
> | Subtract 5 | - | $k + 4$ |
>
> You tell me your answer. I subtract 4 from it to find out what your number is. The manipulation may prove difficult but the idea that k stands for a number is readily established.

From here simple equations such as:

$$m + 7 = 10$$
$$k - 4 = 8$$
$$2p = 32$$
$$r \div 2 = 7$$

can be solved intuitively by answering the questions:

Which number added to 7 gives 10?
Which number with 4 subtracted from it leaves 8?
Which number multiplied by 2 is equal to 32?
Which number divided by 2 is equal to 7?

Stage 3A - Using Concrete Models

There is perhaps a temptation to move directly into inverse operations and get on with equation solving. Many pupils however will benefit from a slower approach which begins with the use of concrete materials. Using a bottletop to represent the unknown number, and seeds (or centicubes or stones) to represent numbers, models of equations may be made as follows.

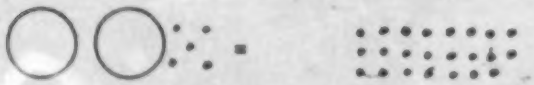
(1)

$x + 4 = 7$

$x = 3$

(2)

$3x + 5 = x + 23$



$3x + 5 = 23$

$2x = 18$

$x = 9$

Check ?

$(3x9) + 5 = 9 + 23$

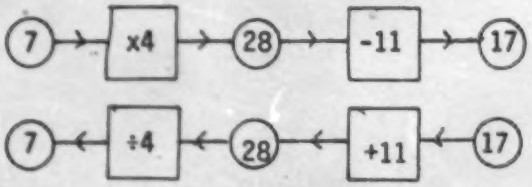
Concrete models are simple where $\times$, $+$ and whole positive numbers are involved. Models for $\div$, $-$ and other numbers are often more difficult than the ideas they are being used to represent. After using models in the simplest cases, a more systematic treatment can be developed using number machines.

Stage 3B - Number Machines and their Inverses

Inverse operations are illustrated well by using practical situations such as the giving of presents.

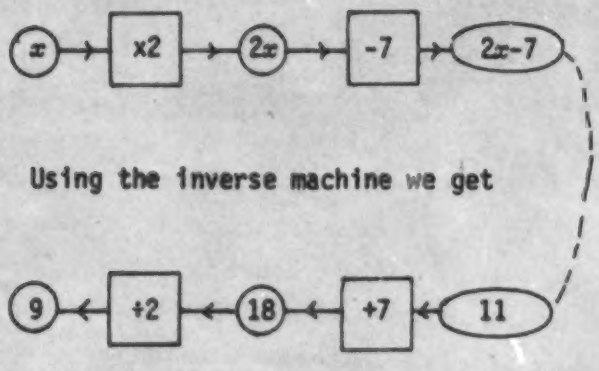
GIVER		RECEIVER
Puts present in box Wraps box with paper Ties string around the paper	↓	Takes present out of box Unwraps paper Unties string ↑

Each action is reversed, and the order of the actions is reversed. The same applies in the case of number machines.



The above example shows a machine and its inverse or "working backwards" machine. Whilst the first machine multiplies by 4 and then subtracts 11, its inverse firstly adds 11 and then divides by 4. The inverse operations ($+$ for $-$ and $\div$ for $\times$) are used and the order of operations is reversed.

Machines can be used to represent equations, e.g., the equation $2x - 7 = 11$ is equivalent to:



Stage 3C - Solving Equations Symbolically

The solution of equations can be abstracted from the method using number machines when pupils are ready for this step. The machines provide the model of inverse operation but this method can be used without the machines. The above example becomes:

$$
\begin{aligned}
2x - 7 &= 11 \\
(2x - 7) + 7 &= 11 + 7 \quad * \\
2x &= 18 \\
(2x) \div 2 &= 18 \div 2 \quad * \\
x &= 9
\end{aligned}
$$

The steps of writing in the inverse operation, shown * above, may soon be omitted.

Stage 3D - Solving Problems

Section 2, above pages 11-24, outlined some of the problems which pupils have with algebra. A major difficulty is the inability to translate a problem into a meaningful algebraic form; this includes the ability to set up an equation, solve it and hence solve the problem. The view expressed here is that the above sequence of development fosters this ability. Further research is required to substantiate this view.

3.2 IMPLEMENTATION

The development of algebra as outlined above is similar to that suggested by the existing syllabuses, except that there is a greater emphasis here

on experimentation and functions, and the use of concrete referents. Whilst the spiral approach to curriculum construction may have its advantages, the algebra content of the syllabuses (detailed in Appendix A) is scattered throughout grades 7-10 in what may appear to many as isolated and unrelated topics. That the syllabus has been modified by the development of pupils texts, to the extent that Units 7.14 and 8.14 have been omitted from grades 7 and 8, is therefore welcomed. The content of these units is included in the sequence of activities described above, and the inclusion of the whole sequence in grades 9 and 10 may be readily achieved as follows.

UNIT 9.1 - NUMBER PATTERNS 3. Replace the activities suggested in syllabus by those recommended for STAGE 1, above.

UNIT 9.6 - NUMBERS AND SYMBOLS 1. Expand this unit to include the activities outlined for STAGE 2, above. The time allocated to this unit may be increased by deferring the work on equations to grade 10.

UNIT 9.12 - EQUATIONS. Defer this unit to grade 10.

UNIT 10.7 - NUMBERS AND SYMBOLS 2. The work on expanding and factoring algebraic expressions, and evaluation by substitution will have already been taught in NUMBERS AND SYMBOLS 1, and need only be revised here. The section on equations, 10.7.6, will now include all that work suggested for STAGE 3, above.

REFERENCES

AF EKENSTAM, A. and NILSSON, M. 'A New Approach To The Assessment of Children's Mathematical Competence', *Educational Studies in Mathematics*, 10(1), 41-66, 1979.

ANDERSON, J. *The Mathematics Curriculum - Algebra* Blackie, Glasgow, 1978. 1978.

BRITT, M. 'Papua New Guinea: Secondary Mathematics - A Curriculum Development Project', paper presented to the Fourth International Congress on Mathematical Education, Berkeley, 1980.

DAVIS, R.B., JOCKUSCH, E. and McKNIGHT, C. 'Cognitive Processes in Learning Algebra,' *Journal of Children's Mathematical Behaviour*, 2(1), 85-113, 1978.

DIENES, Z.P. *Building Up Mathematics* Hutchinson, London, 1964 (a).

DIENES, Z.P. *An Experimental Study of Mathematics Learning* Hutchinson, London, 1964 (b).

DIEUDONNÉ, J. 'Abstraction in Mathematics and the Evolution of Algebra' in W.E. Lamon (ed.) *Learning and the Nature of Mathematics*, Science Research Associates, Chicago, 1972.

FITZGERALD, A. 'School Mathematics and Employment at 16+ With Particular Reference to Commercial Work', *Mathematics in Schools*, 9(3), 26-31, 1980.

GILES, G. *Operations 1 - Flags; Operations 2 - Pattern and Notation; Number Mappings 1 - Number Machines; Number Mappings 2 - Mappings and Graphs; Number Patterns 1 - Simple Mappings; Number Mappings 2 - Further Mappings*, Oliver and Boyd, 1979.

HART, K.M. *Secondary School Children's Understanding of Mathematics*, Research Monograph, Centre for Science Education, Chelsea College, London, 1980.

HAYTER, R.J. 'The High School Mathematics Curriculum in Papua New Guinea - An Evaluation Study', Research Monograph, School of Education, University of Bristol, 1980.

INSTONE, S. 'The SMP Algebra Project', paper presented to the Fourth International Congress on Mathematics Education, Berkeley, 1980.

JONES, P.L. 'Quality in Education and its Relationship to Mathematics Achievement', paper presented to the Conference of Principals and Directors of Post Secondary Education and Training Institutions, Waigani, 1980.

KÜCHEMANN, D. 'Children's Understanding of Numerical Variables', *Mathematics in Schools*, 7(4), 23-26, 1978.

LANCY, D.F. 'The Indigenous Mathematics Project' *Papua New Guinea Journal of Education*, 14 (Special issue), 1978.

LAYCOCK, M. and SCHADLER, R.A. *Algebra in The Concrete*, Activities Resources Co., Hayward, 1973.

LEAN, G.A. *Mathematics Learning Project - Progress Report 1975*, The Papua New Guinea University of Technology, Lae, 1975.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS, *An Agenda for Action*, Reston, 1980.

NORRIS, E.T. *Mathematics for the Majority - Algebra of a Sort*, Chatto and Windus, St. Albans, 1973.

ROAKEINA, S.G. 'Report of the Committee of Enquiry into Standards of High School Students Entering Colleges', Department of Education, Waigani, 1977.

ROBERTS, R.E. 'Primary Mathematics Learning', unpublished M.Ed. thesis, University of Papua New Guinea, 1981.

SOUVINEY, R.J., KADA, V. and MALAGA, D, 'The Indigenous Mathematics Project of Papua New Guinea', paper presented to the Fourth International Congress on Mathematical Education, Berkeley, 1980.

SOUVINEY, R.J. *Problem Solving Kids Care About*, Goodyear, Los Angeles, 1981.

SULLIVAN P. 'The Standards Debate: An Alternative Strategy', *Mathematics Education Centre Report No. 15*, Papua New Guinea University of Technology, 1981.

UNESCO - UNDP, *Curriculum Development - The South Pacific*, Curriculum Development Unit, University of the South Pacific, 1976.

UNESCO - UNICEF, *The Development of Science and Mathematics Concepts in Young Children in African Countries*, University of Sussex, 1974.

APPENDICES

A: Summary of the Algebra Content of P.N.G. Grades 7-10 Syllabuses 42

B: Questionnaire for Mathematics Subject Masters 45

C: Algebra Survey 50

D: List of Institutions 53

E: Algebra Test 54

F: Marking Code 60

G: Results of Algebra Test 62

H: Classification of Algebra Test Items 63

APPENDIX A

SUMMARY OF THE ALGEBRA CONTENT OF P.N.G. GRADES 7-10 SYLLABUSES

The syllabus objectives are summarized below. The notation 9.6.4 refers to GRADE 9, UNIT 6, OBJECTIVE 4.

THE PROVINCIAL HIGH SCHOOL SYLLABUS (GRADES 7-10)

7.14 Number Problems (Suggested teaching time 1 week.)

Introduction

This unit provides a first introduction to algebra. The use of a number machine is suggested in order that the ideas not be treated in a too abstract manner from the beginning. Pronumerals, letters or symbols standing in place of numbers, should be introduced as a convenient shorthand method of writing number sentences.

7.14.1 Identify the output, input or rule in a variety of number sentences in which two of the above are given.

8.14 Problems and Relations (Suggested teaching time 2-3 weeks.)

Introduction

This unit is an extension of 7.14. Most things found in our environment are related to each other in some way. Whether it be in terms of their physical separation; nearness, remoteness etc., or in terms of some other property they possess. The study of relations is fundamental to the study of mathematics.

8.14.1 Solve simple number problems related to activities with number machines.

8.14.2 Discover relationships between sets of numbers, and to write relations as sets of ordered pairs.

8.14.3 Illustrate simple relations on coordinate graphs.

9.6 Numbers and Symbols 1 (suggested teaching time 3-4 weeks.)

Introduction

Pupils are already using formulas (area, volume) but have not yet learnt to write these formulae in short precise mathematical ways. In this unit we learn how to use letters (standing for numbers, measurements or other things) to help us work out answers to problems that are often expressed in lengthy English sentences.

9.6.1 Use of brackets in arithmetic expressions.

9.6.2 Use of mathematical shorthand to write English sentences. (3 bananas + 2 bananas = 5 bananas : $3b + 2b = 5b$). Addition and subtraction of like terms. Unlike terms.

9.6.3 Write mathematical formulas in shorthand notation. (Area of rectangle = base x height : $A = b \times h$)

9.6.4 Use formulas to work out areas and volumes.

9.6.5 Evaluate algebraic expressions by substitution.

This work is continued in UNIT 9.8 - CIRCLES 2 and UNIT 9.11 - VOLUME 2 where formulas are used in the calculation of circumference and area of circles, and in the calculation of the volume of various solids.

9.12 Equations (Suggested teaching time 1-2 weeks.)

Introduction

Problem solving is a fundamental mathematical activity. Before a problem can be solved it has to be put into mathematical terms. In this Unit we learn how to solve algebraic equations and practice solving simple problems.

9.12.1 Intuitive solution of simple number sentences involving whole numbers, decimals and fractions.

9.12.2 Write simple English sentences as mathematical sentences.

9.12.3 Solve linear equations involving not more than two steps. It is strongly recommended that pupils use the flowchart method.

9.12.4 Solve simple word problems involving linear equations.

Symbols are used sparingly in UNIT 10.3 - RATIO and UNIT 10.5 - SOCIAL MATHS (Simple interest formula I = PXTXR).

10.7 Numbers and Symbols 2 (Suggested teaching time 3-4 weeks.)

Introduction

This Unit extends the algebraic work that was begun in UNITS 9.6 and 9.12.

10.7.1 Use of powers in numerical and algebraic expressions.

10.7.2 Simplify simple algebraic expressions using the index laws.

10.7.3 Expand algebraic expressions. $((x+2)(x+3) = x^2 + 5x + 6)$

10.7.4 Factor simple algebraic expressions $(pq + 3p = p(q + 3))$

10.7.5 As 9.6.5 - Evaluate algebraic expressions by substitution.

10.7.6 As 9.12.3 - Solve simple linear equations.

10.7.7 As 9.12.4 - Solve simple word problems that involve equations.

Symbols are also used in UNIT 10.9 - PYTHAGORAS, and in UNIT 10.10 - TRIGONOMETRY.

APPENDIX B

SECTION A

Questionnaire for Mathematics Subject Masters

Grade 9

School _____

Subject Master's Name _____

Instructions

1. Please refer to Grade 9 syllabus.
2. Mark boxes with ✓.

Unit Title	Objective	Planning and preparing lessons to achieve objective is			Pupils achieve objective with		
		not difficult	moderately difficult	difficult	no difficulty	some difficulty	great difficulty
Number Patterns	9.1.1						
Decimals and Percentages	9.2.1						
	9.2.2						
	9.2.3						
	9.2.4						
	9.2.5						
	9.2.6						
	9.2.7						
	9.2.8						
	9.2.9						
	9.2.10						
	9.2.11						
Properties of Shapes	9.3.1						
	9.3.2						
	9.3.3						
Area	9.4.1						
	9.4.2						
	9.4.3						
	9.4.4						
	9.4.5						

Fractions	9.5.1						
	9.5.2						
	9.5.3						
	9.5.4						
	9.5.5						
	9.5.6						
	9.5.7						
	9.5.8						
Numbers and Symbols	9.6.1						
	9.6.2						
	9.6.3						
	9.6.4						
	9.6.5						
Rates and Graphs	9.7.1						
	9.7.2						
	9.7.3						
Circles	9.8.1						
	9.8.2						
	9.8.3						
Statistics	9.9.1						
	9.9.2						
	9.9.3						
	9.9.4						
	9.9.5						
	9.9.6						
Constructing Shapes and Solids	9.10.1						
	9.10.2						
Volume	9.11.1						
	9.11.2						
	9.11.3						
	9.11.4						
Equations	9.12.1						
	9.12.2						
	9.12.3						
	9.12.4						

SECTION B

1. It is seldom a good idea to teach a topic such as directed numbers in one lump over an extended period of time.

   For this reason it is proposed to include some introductory work on directed numbers in grade 9 rather than leave it all to grade 10.

   Please mark with a ✓ your reaction to this proposal.

Strongly	Agree	No comment	Disagree	Strongly disagree

2. List any topics that you feel should be included in grade 9, but which are not in the grade 9 syllabus.

   Please give reasons.

Topic	Reason for inclusion in grade 9

3. List any topics that you feel should be excluded from grade 9.

Please give reasons.

Topic	Reason for exclusion from grade 9

4. Would you and other grade 9 teachers at your school be willing to fully participate in the trialling programme for grade 9 material?

Participating in such a programme would necessitate:

(a) writing reports on curriculum material with regard to relevance, appropriateness, language usage etc.

(b) being observed teaching the trial materials to grade 9 pupils.

(c) suggesting ideas to assist in the development of suitable curriculum materials.

Please mark box with a ✓.

Not willing to participate in curriculum trialling ☐

Willing to participate in curriculum trialling ☐

If you are willing, you may wish to add some comments on the page provided.

Comments

APPENDIX C

U P N G ALGEBRA SURVEY

During 1980 the secondary mathematics curriculum underwent a close examination at both the Provincial High School and National High School level. Information from both universities indicates that the major area of mathematical weakness of Preliminary Year and Foundation Year students is Algebra. Texts in the form of pupil's booklet are now in use in grades seven and eight, and work is soon to begin on materials for grade nine. The current mathematics syllabuses detail a development of algebra which starts at grade nine (with the use of symbols and the solution of simple equations) and continues as a major topic in grades ten, eleven and twelve. Research to identify levels of cognitive demand, appropriate content sequencing, and teaching methods is needed before suitable curriculum materials can be developed for the Papua New Guinean learner.

There has also been widespread reports from employers and post-secondary institutions about the mathematical shortcomings of the present grade ten leavers. Students are not proficient in such topics as operations with decimals, fractions, ratio, proportion, percentages, the metric system, estimation, and so on.

The present grade nine and ten mathematics syllabuses allocate approximately ten percent of the content to the use of symbols, substitution in formulae and solution of equations. This survey is an attempt to find out what algebra is useful to grade ten leavers.

Please complete the attached QUESTIONNAIRE and return to:

MIKE SIFORD
TEACHING METHODS AND MATERIALS CENTRE
BOX 4820
UNIVERSITY POST OFFICE.

INSTITUTION _____

NAME OF COURSE(S) _____

Please ✓ one of the three boxes, essential/useful/not required, for each of the items listed below.
(* asks you to provide an example if this item is appropriate)

	This CONTENT is			
	ESSENTIAL	USEFUL	NOT REQUIRED	
1. - MANIPULATION OF SYMBOLS				
(a) Collecting like terms				$2a+5a - a - 2b$
(b) multiplying out brackets				$4(x-3)$ = $(x-2)(x+5)$=
(c) factoring expressions				$15p-25mp$ = $x^2-10x+25$ =
(d) algebraic fractions				$\frac{x}{x+2} - \frac{3}{x^2-4}$ =
2. - USE OF THE LAWS OF INDICES				
(a) Positive indices				$2^7 \times 2^9$ =
(b) negative indices				$x^2 \div x^{-5}$ =
(c) fractional indices				$\sqrt{16^{4x}}$ =
3. - USE OF FORMULAE				
(a) simple substitution				$A = 3.14R^2$ find A given $R = 7.2$ cm
(b) substitution requiring more difficult manipulation.				$V = 3.14R^2h$ find R given $V=100m^3$ and $h=6m$
4. - SOLUTION OF EQUATIONS				
(a) to solve a linear equation				$2(x+4) = 18$
(b) to solve a quadratic equations				$x^2 - 7x+10 = 0$
(c) to solve simultaneous equations				$2x-y = 8$ $x+y = 1$
(d) to set up an equation given a problem in the work situation.				*
5. * (list any other topics which involve algebra.)				*

Algebra <u>should</u> be included in the grades nine and ten syllabuses.
Please mark with a ✓ your reaction to this statement.

STRONGLY AGREE	AGREE	NO COMMENT	DISAGREE	STRONGLY DISAGREE

ADDITIONAL COMMENTS

APPENDIX D

LIST OF INSTITUTIONS

The following institutions responded to the questionnaire shown at Appendix C.

Burns Philp.

Department of Works and Supply Training Centre, Lae.

Elcom Training College.

Sepik Agricultural College.

Civil Aviation Training College.

Institute of Administrative Studies.

Department of Primary Industry.

* Port Moresby Technical College.

P.N.G. Defence Force Training Depot - Goldie Barracks.

P.N.G. Defence Force Health Centre.

P.N.G. Defence Force Air Transport Training Centre.

P.N.G. Defence Force Patrol Boat Base, Lombrum.

* verbal response only; all others completed the questionnaire.

APPENDIX E

PLEASE PRINT

UPNG ALGEBRA SURVEY
TIME: 1 HOUR

NAME .....

SCHOOL .....P.H.S./N.H.S.

AGE ..... YEARS .....MONTHS

Practice question 1 What number does $a + 4$ stand for if $a = 2$ ....
What number does $4a$ stand for if $a = 2$ ....

Practice question 2

$x \to 3x$	$k \to k + 8$	$m \to m - 4$
$2 \to 6$	$4 \to 12$	$10 \to$ ....
$5 \to$	$7 \to$	$x \to$ ....

NOW CHECK YOUR ANSWERS AGAINST THE ANSWERS ON THE BACK PAGE.

1. Fill in the missing numbers

$x \to x + 2$	$t \to 4t$
$6 \to$	$3 \to$ ....
$r \to$ I1(67.3) L.O.	$x - 1 \to$ I2(43.3) L.O.

2. $a + 3a$ can be written more simply as $4a$.
Write these more simply, where possible:

$2a + 5a =$ I3(97.3) L.O.	$3a - (b + a) =$ I8(71.6) L.O.
$2a + 5b =$ I4(72.0) L.O.	$a + 4 + a - 4 =$ I9(95.6) L.O.
$(a + b) + a =$ I5 (72.0) L.O.	$(a + b) + (a - b) =$ I10(78.3) L.O.
$2a + 5b + a =$ I6(89.3) L.O.	$(a + b) - (a - b) =$ I11(66.7) L.O.
$(a - b) + b =$ I7(70.7) L.O.	$a - b - 2a - 3b =$ I12(74.3) L.O.

3. Add 4 onto each of these: 8 $n + 5$ $3n$
.... I13(77.6) L.O. I14(76.7) L.O. + A.L.C.

4. Multiply each of these by 4: 8 $n + 5$ $3n$
.... I15(88.7) L.O. I16(93.0) L.O.

5. Remove brackets and simplify, where possible:

$(x - 4)(x + 4)$	$(2m - 3)^2$	$(2x - y)(x - 3y)$
=	=	=
= I17(94.9) L.O.	= I18(75.6) L.O.	= I19(85.0) L.O.

6. If $a + b = 43$ then $a + b + 2$ = I20(98.0) L.I.

If $n - 246 = 762$ then $n - 247$ = I21(73.3) L.I.

If $e + f = 8$ then $e + f + g$ = I22(63.3) L.O. + A.I.C.

7. Write down the smallest and largest of these:

	smallest	largest
$n + 1$, $n + 4$, $n - 7$, n, $n - 3$	I23 (82.0)	L.O.

8. Which is the larger $2n$ or $n + 2$? I24(13.4) L.C.

Explain: .....

9. $a = b + 3$. If b is increased by 2, by how much does a increase?

I25(61.3) L.C.

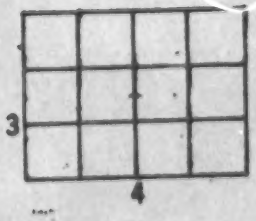
$f = 3g + 1$. If g in increased by 2, by how much does f increase?

I26(18.0) L.C.

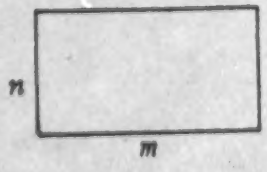
$t = 17 - 2k$. If k is increased by 2, what happens to t ?

I27(27.3) L.C.

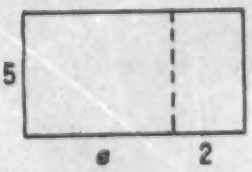
10. What are the areas of these shapes?



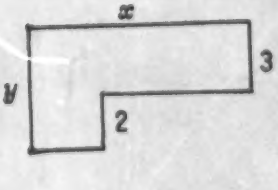
A = ...



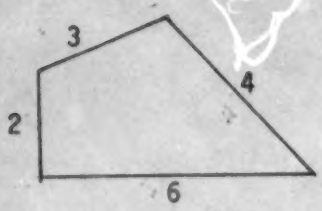
A = I28(71.3) L.C.



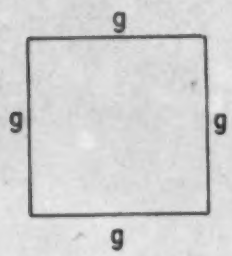
A = I29(52.7) L.C.



A = I30(36.0) L.C.

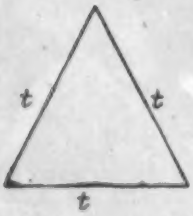
11. 

The perimeter of this shape is equal to $2 + 3 + 4 + 6$, which equals 15.

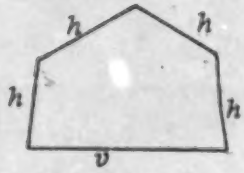


This square has sides of length g.
So for its perimeter we can write $P = 4g$.

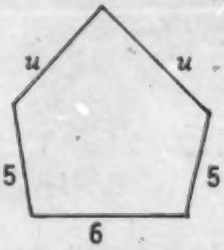
What can we write for the perimeter of each of these shapes?



P = I31(92.0)L.C.



P = I32(81.3)L.C.



P = I33(76.3)L.C.

Each side is of length 2.

Part of this figure is not shown. There are n sides altogether.

P = I34(22.7)L.C.

12. Factorize: $am + bn - mb - an$

= I35(15.9) L.O. + G.O.

$y^2 - 2y - 15$

= I36(80.7) L.O.

13. What can you say about a if $a + 5 = 8$? I37(98.0) L.E.

What can you say about b if $b + 3$ is equal to $3b$? I38(5.3) L.O.E.

14. Solve the equation $2(x + 6) - 3(x - 4) = 1$

x = I39(58.0) L.O.E.

15. What can you say about m if $m = 3n + 1$
and $n = 4$ I40(90.7) L.E.

What can you say about r if $r = s + t$
and $r + s + t = 30$ I41(30.0) L.O.E.

What can you say about c if $c + d = 10$
and c is less than d I42(30.3) L.O.E.

16. Make t the subject of the formula $V = u - gt$.

$t =$ I43(78.6) L.O.

17. Simplify: $\frac{2}{x+1} - \frac{x}{x^2-1}$

I44(44.7) L.O. + L.E.S.

18. If John has J pineapples and Tau has T pineapples, what could you write down for the number of pineapples they have altogether?

I45(74.6) L.C.

19. In a shape like this you can work out the number of diagonals by taking away 3 from the number of sides.

5 SIDES
2 DIAGONALS

So a shape with 5 sides has 2 diagonals;
a shape with 47 sides has ..... diagonals;
a shape with k sides has I46(58.0) L.C. diagonals.

20. Pineapples cost P toea each and water-melons cost W toea each.
If you buy 3 pineapples and 4 water-melons, what does
$3P + 4W$ stand for? I47(65.3) L.C.
.....

Cabbages cost 40 toea each and pumpkin 30 toea each.
If you but C cabbages and P pumpkin,
What is the total number of vegetables bought? I48(36.7) L.C.
.....
What is the total price paid for the vegetables? I49(40.7) L.C.
.....

21. Michael's basic wage is K30 per week.
He is also paid another K2 for each hour of overtime that he works.

Calculate Michael's total wage for a week in which he worked 4 hours
of over-time. I50(77.3) N K70, K86 (10.0)
.....

If h stands for the number of hours of overtime that he works, and
if W stands for his total wage in kina, write down an equation
connecting W and h.
.....I51(13.3) L.C.
.....

22. When are the following true - always, never or sometimes?
UNDERLINE THE CORRECT ANSWER

$x + y + z = z + x + y$ Always Never Sometimes, when I52(82.7)L.O.
.....
$l + m + n = l + p + n$ Always Never Sometimes, when I53(3.3) L.C.
.....

23. The sentence $(x + 1)^3 + x = 349$ is TRUE when $x = 6$.
What value of x will make the sentence $(5x + 1)^3 + 5x = 349$ TRUE ?
$x =$ I54(11.0) L.C.E. or L.O.E. (0.0)
.....

24. The POST COURIER costs 12 toea and the NEW NATION MAGAZINE costs 50 toea. You buy some newspapers and mazagines and altogether is costs K5.30.

If you bought P POST COURIERS and n NEW NATION MAGAZINES for K5.30, what can you write down about p , n and 530?

I55(32.0) L.C.

25. Fill in the missing value of y in this table.

x	y
4	11
5	13
8	19
10	I56(51.3)N
25	53

Write down an equation which expresses y in terms of x .

y = I57(30.0) L.C.

Practice Questions

ANSWERS What number does $a + 4$ stand for if $a = 2$ 6

What number does $4a$ stand for if $a = 2$ 8

$x \rightarrow 3x$	$k \rightarrow k + 8$	$m \rightarrow m - 4$
$2 \rightarrow 6$	$4 \rightarrow 12$	$10 \rightarrow 6$
$5 \rightarrow 15$	$7 \rightarrow 15$	$x \rightarrow x - 4$

APPENDIX F

	MARKING CODE						
ITEM	1	2	3	4	5	6	7
1	$r + 3$			$r3$	$r-x+2; T+4$	2,10, ...	
2	$4x - 4$	$4(x-1)$			$-4xt;$	-4,	$x-4$, $4x-1$
3	$7a$						
4	$2a+5b$			$2a5b$	$7ab$		
5	$2a+b$			$2ab$	$ab + a$		$a^2 + ab$
6	$3a + 5b$			$3a5b$	$8ab$, $3a+b$		
7	a				$ta+2b$		$ab - 1^2$
8	$2a - b$		$3a-b-a$		$3a-b$, $4a-b$		$3ab - b^2$
9	$2a$		$2a+4-4$		$2a+8$, $8a$		
10	$2a$		$2a+b-b$		$2a+b$, 0		a^2-b^2
11	$2b$				$2a+2b$, 0		$-a^2+ b^2$
12	$-a-4b$		$-a-4$		$a-4-b$		
13	$n+9$		$n+5+4$		$8n, 4n+9, n+13$	9 , 13	
14	$3n+4$				$4n+7$	12	$7n$
15	$4n+20$		$4(n+5)$		$4n+9; 20n$	$9; 12+20$	$4n+5; n+20$
16	$12n$		$3n \times 4$	$12n(n^2)$	$24n$, $48n$	12	
17	x^2-16	$x^2 - 4^2$	$x^2-4x+4x+16$		$x^2-4, x=4$		
18	$4m^2-12n+9$		$4m^2-6n-6n+9$		$4m^2+9$		
19	$2x^2-7xy+3y^2$	Copying error	$2x^{2}7xy^{2}3y^2$				
20	45						
21	781				763		
22	$8 + 9$			$8g$	0	10,11,12,24,15	
23	$n-7, n+4$		$n+4, n-7$		$n-3, n$ etc.		
24	Neither	poor expl.			$2n$, $n+2$	n any no. not fixed	
25	2				1, twice	5	
26	6		6 times		2	7	
27	dec.by 4		decreases		2	13	
28	mn			$mn^2; (mn)^2$			
29	$5e+10$	$5(e+2)$	$e+2=5$	$5(e+2)^2$	$10e$	25,35	
30	$xy-6$			xy^2-6^2	$xy; x+y-5$	10	
31	$3t$	$t+t+t; 2t+t$ $v+h+h+h+h$			$\frac{3t}{2}$, 3^2		
32	$V+4h$	$v+2h+2h$		$4hv$	$4h \times v$		
33	$2u+16$	$2u+10+6$	$(5)2+6+2u$		$2u+11, 300u$		
34	$2n$				$8+n$, $8+4n$	14,16,18 ...	
35	$(a-b)(m-n)$		$m(a-b)+n(a-b)$				
36	$(y-5)(y+3)$		$(y+5)(y-3)$		$(y-5)(y-3)$		
37	$a = 3$		$a=8-5$				
38	$b = 1.5$						
39	$x = 23$		$x = -23$		-1 etc.		

	MARKING CODE						
ITEM	1	2	3	4	5	6	7
40	$m = 13$				$m = 12$		
41	$r = 15$	$2r = 30$	$r = 30-r$				
42	$c < 5$	$c = 1,2,3,4$	$c = 2;4$				
43	$t = \frac{u-v}{g}$	$t = \frac{v-u}{-g}$	$\frac{v-u}{g}$; $\frac{v+u}{-g}$				
44	$\frac{x-2}{x^2-1}$	$\frac{2(x-1)-x}{(x-1)(x+1)}$	$\frac{x^2-x-2}{(x^2-1)(x+1)}$				
45	$J + T$	J pineapples $+T$ pineapples		$J+T = JT$	JT		
46	$k - 3$					27 etc	H
47	Total cost	3 pine for P +dw-m for v	toea toea.		3 Pine + dw-m were bought		
48	$C + P$			$C+P=CP$	CP	2	
49	$(40c+30p)$ toea			$40c+30P=70CP$		70 toea	
50	K 38				K70, K86		
51	$W = 30+2h$	$30+2h$	$W=30+h$ $W=30+4h$			Answer using K38, K70, K86	
52	Always						
53	Sometimes		Sometimes				
	$m = P$		?		Never		
54	$x = 1,2$		$x = \frac{x}{5}$		0, Expand		
55	$12p+50n=530$		$12p+50p=530$ $12p+50n=5.30$		$p+n = 530$ $p+n = 5.30$		
56	28				22 etc.		
57	$y=2x+3$	$y=x+x+3$			$x-8$ etc.		

CODE	DESCRIPTION
1	Correct.
2	nearly correct.
3	correct but incomplete, or minor error.
4	ambiguous - lack of, or oversupply of, operations.
5	common wrong answers (see code 7).
6	wrong answers obtained by replacing a letter by a number.
7	common wrong answers for special mention.
8	not attempted.
9	not attempted probably because of lack of time.

APPENDIX G

ALGEBRA SURVEY: UPNG PRELIMINARY YEAR - OCTOBER 1980: N = 150

	MARKING CODE								
ITEM	1	2	3	4	5	6	7	8	9
1	67.3	-	-	8.0	4.7	7.3	-	12.7	-
2	34.0	9.3	-	-	22.0	12.7	6.7	15.3	-
3	97.3	-	-	-	1.3	-	-	1.3	-
4	72.0	-	-	2.0	15.3	-	-	10.7	-
5	72.0	-	-	2.7	9.3	-	13.3	2.7	-
6	89.3	-	-	2.0	7.3	-	-	1.3	-
7	70.7	-	-	-	11.3	-	14.7	3.3	-
8	71.3	-	0.7	-	17.3	-	5.3	4.7	-
9	95.3	-	0.7	-	2.7	-	-	1.3	-
10	78.0	-	0.7	-	6.7	-	11.3	3.3	-
11	66.7	-	-	-	19.3	-	8.7	5.3	-
12	74.0	-	0.7	-	20.7	-	-	4.7	-
13	77.3	-	0.7	-	18.7	2.7	-	0.7	-
14	76.7	-	-	-	2.7	1.3	18.7	0.7	-
15	88.0	-	1.3	-	4.0	2.0	4.0	0.7	-
16	92.0	-	2.0	0.7	3.3	1.3	-	0.7	-
17	93.3	1.3	0.7	-	4.7	-	-	-	-
18	73.3	-	4.7	-	21.3	-	-	0.7	-
19	78.1	0.7	11.3	-	6.0	-	-	3.3	-
20	98.0	-	-	-	-	-	-	2.0	-
21	73.3	-	-	-	25.3	-	-	1.3	-
22	63.3	-	-	2.7	1.3	25.3	-	7.3	-
23	80.0	-	4.0	-	15.3	-	-	0.7	-
24	8.7	4.7		-	86.0	0.7	-	-	-
25	61.3	-	-	-	17.3	13.3	-	8.0	-
26	17.3	-	1.3	-	56.0	14.7	-	10.7	-
27	5.3	-	44.0	-	30.0	9.3	-	11.3	-
28	71.3	-	-	25.3	2.7	0.7	-	-	-
29	32.7	18.0	4.0	13.3	16.7	10.7	-	4.7	-
30	36.0	-	-	8.7	27.3	12.7	-	15.3	-
31	89.3	2.7	-	-	6.0	-	-	2.0	-
32	78.0	3.3	-	3.3	12.0	-	-	3.3	-
33	66.7	5.3	12.7	-	14.0	-	-	1.3	-
34	22.7	-	-	-	28.0	42.7	-	6.7	-
35	7.3	-	17.3	-	28.7	-	-	46.7	-
36	78.7	-	4.0	-	8.7	-	-	8.7	-
37	96.7	-	2.7	-	0.7	-	-	-	-
38	5.3	-	-	-	71.3	-	-	23.3	-
39	52.7	-	10.7	-	36.7	-	-	-	-
40	90.7	-	-	-	7.3	-	-	2.0	-
41	28.7	0.7	1.3	-	48.7	-	-	20.7	-
42	9.3	10.0	22.0	-	38.0	-	-	20.7	-
43	30.0	41.3	14.7	-	14.0	-	-	-	-
44	40.7	3.3	1.3	-	46.7	-	-	8.0	-
45	69.3	5.3	-	10.0	10.0	-	-	5.3	-
46	58.0	-	-	-	16.0	8.7	4.7	12.7	-
47	44.0	21.3	-	-	28.0	-	-	5.3	1.3
48	36.7	-	-	2.7	23.3	31.3	-	4.7	1.3
49	40.7	-	-	2.0	5.3	45.3	-	4.7	2.0
50	77.3	-	-	-	19.3	-	-	1.3	2.7
51	11.3	0.7	2.7	-	60.7	11.3	-	9.3	4.0
52	82.7	-	-	-	8.0	-	-	2.7	6.7
53	6.0	-	4.7	-	78.7	-	-	3.3	7.3
54	10.7	-	0.7	-	56.0	-	-	20.0	12.7
55	23.3	-	17.3	-	27.3	4.0	-	11.3	16.7
56	51.3	-	-	-	25.3	-	-	4.7	18.7
57	29.3	0.7	-	-	34.7	-	-	-	35.3

APPENDIX H

THE CLASSIFICATION OF THE ITEMS

ITEM	CLASS	DIFFICULTY
1	L.O.	67.3
2	L.O.	43.3
3	L.O.	97.3
4	L.O.	72.0
5	L.O.	72.0
6	L.O.	89.3
7	L.O.	70.7
8	L.O.	71.6
9	L.O.	95.6
10	L.O.	78.3
11	L.O.	66.7
12	L.O.	74.3
13	L.O.	77.6
14	L.O. (A.L.C.)	76.7
15	L.O.	88.7
16	L.O.	93.0
17	L.O.	94.9
18	L.O.	75.6
19	L.O.	85.0
20	L.I.	98.0
21	L.I.	73.3
22	L.O. (A.L.C.)	63.3
23	L.I.	82.0
24	L.C. (G.N.)	13.4
25	L.C. (G.N.)	61.3
26	L.C. (G.N.)	18.0
27	L.C. (G.N.)	27.3
28	L.C.	71.3
29	L.C.	52.7
30	L.C.	36.0
31	L.C.	92.0
32	L.C.	81.3
33	L.C.	76.3
34	L.C.	22.7
35	L.O. (G.O.)	15.9
36	L.O.	80.7
37	L.E.	98.0
38	L.O.E.	5.3
39	L.O.E.	58.0
40	L.E.	90.7
41	L.O.E.	30.0
42	L.O.E. (G.N.)	30.3
43	L.O.	78.6
44	L.O. (L.E.S.)	44.7
45	L.C.	74.6
46	L.C.	58.0
47	L.C.	65.3
48	L.C.	36.7
49	L.C.	40.7
50	N.	77.3
5.	L.C.	13.3
52	L.O.	82.7
53	L.O. (G.N.)	8.3
54	L.C.E.	11.0
55	L.C.	32.0
56	N.	51.3
57	L.C.	30.0

CLASS	NUMBER OF ITEMS	SPEAD	MEAN DEGREE OF DIFFICULTY
L.E.	2	90.7-98.0	94.4
L.I.	3	73.3-98.0	84.4
L.O	25	15.9-95.6	74.2
L.O.E.	4	5.3-58.0	30.9
L.C.	20	8.3-92.0	45.6
L.C.E.	1	-	11.0
NUMERICAL	2		
TOTAL	57		

G.O. = GOAL ORIENTATION

A.L.C. = ACCEPTANCE OF LACK OF CLOSURE

G.N. = GENERALIZED NUMBER

L.E.S. = LONG EXECUTION SEQUENCE

The following M.E.C. Reports have been published:

No. 1: A. EDWARD 'Report for 1978, including a Short History of the Mathematics Learning Project and Mathematics Education Centre up to 1977', 1978.

No. 2: A.J. BISHOP 'Spatial Abilities in a Papua New Guinea Context', 1978.

No. 3: C. MEEK and N. FERIL 'A Pilot Study into the Level of Comprehension of Some Words taken from the Papua New Guinea Community Schools Mathematics Syllabus', 1978.

No. 4: A.J. BISHOP 'Developing Spatial Abilities', 1977.

No. 5: A. EDWARDS 'A Study of the Issues Involved in Establishing the Mathematics Education Centre and a Report on its Work from December 1978 to May 1979', 1979.

No. 6: A. EDWARDS and A. BAJPAI 'Preliminary Report on the Basic Arithmetic Test Carried out in Post-Secondary Institutions in Papua New Guinea in 1979', 1979 (out of print).

No. 7: A. EDWARDS 'Final Report on the Establishment of the Mathematics Education Centre', 1979 (out of print).

No. 8: G.A. LEAN 'The Training of Spatial Abilities: A Bibliograph', 1980.

No. 9: G.A. LEAN 'An Investigation of Spatial Ability Among Papua New Guinea Students', 1980.

No. 10: G.A. LEAN and M.A. CLEMENTS 'Spatial Ability, Visual Imagery and Mathematical Performance', 1980.

No. 11: P.L. JONES 'The Introductory Mathematics Programme at the Papua New Guinea University of Technology - A Survey of Student Opinion', 1980.

No. 12: P.L. JONES 'A Study of Proportional Reasoning', 1980.

No. 13: M.A. CLEMENTS and G.A. LEAN 'Influences on Mathematical Learning in Papua New Guinea: Some Cross - Cultural Perspectives' 1981, (2nd edition).

No. 14: M.A. CLEMENTS 'Careless Errors Made by Sixth-Grade Children on Written Mathematical Tasks', 1981 (2nd edition).

No. 15: P. SULLIVAN 'The Standards Debate: An Alternate Strategy', 1981.

No. 16: P.L. JONES 'Mathematical Language and the Learning of Mathematics in Papua New Guinea', 1981.

No. 17: F. AUSBURN and L. AUSBURN 'Perception, Imagery and Education in Developing Countries', 1981.

No. 18: M.A. CLEMENTS and P.L. JONES 'The Education of Atawe', 1981.

No. 19: P.L. JONES 'Learning Mathematics in a Second Language: A Problem with More and Less', 1981.

No. 20: M. SIFORD 'A Study of Algebra in the Provincial High Schools Curriculum', 1981.

The follow M.E.C. Technical Reports have been published:

No. 1: P. SULLIVAN 'Report of the First Conference to Prepare Mathematics Material for Post-Secondary Courses', 1981.

No. 2: P. CLARKSON 'Report of the 1980 National High School Year 12 Mathematics Marking Panel'.

Other publications include:

P. SULLIVAN 'An Introduction to Nursing Mathematics'. (K1.50)

P. CLARKSON (ed) 'Research in Mathematics Education in Papua New Guinea, 1981', Proceeding of the First National Conference on Mathematics Education, May, 1981.